Robust H_∞ Fault-Tolerant Control for Dynamic Positioning Ships based on Sampled-data

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Abstract: This study addresses the robust H_∞ fault-tolerant control problem for dynamic positioning (DP) ships based on sampled-data with actuator fault. First, the actuator-failure mode is established. Then, input delay approach combined with Lyapunov-Krasovskii functional is used to determine the system's asymptotical stability and achieve H_∞ tracking performance. Then H_∞ fault-tolerant sampled-data controller is obtained by means of linear matrix inequality (LMI). Finally, simulation results show that the designed controller for DP ships is effective so that the output of sampled-data DP ships system can track the reference signal without steady-state error in the existence of varying environment disturbance and actuator faults.

Keywords: dynamic positioning ships, sampled-data control, actuator fault, fault-tolerant control

1. INTRODUCTION

The ship DP system, which is controlled by a computer, can keep a ship's desired position and heading by using active thrusters (see T. I. Fossen, 2002), and it has been employed in wide range of ship types such as offshore support vessels, Semi-submersible crane vessels, cruise ships, oceanographic research vessels and so on. In the past few years, the DP ships technology has become a hot research topic, and several results have been reported for DP system (see Strand et al., 1999; Zhao et al., 2010; Wang et al., 2014; Li et al.,2014 and Du et al.,2016), Such as proportional integral derivative (PID) (see Fossen, 1994); Kalman filtering technology (see Balchen et al. , 1976; A. J. Sørensen et al., 1996 and Xie, 2014); Vectorial backstepping techonology (see Fossen et al., 1998 and Snijders, 2005) ; passive nonlinear observer (see Fossen et al.1999; Lin et al., 2013; Wang, 2012 and Lindegaard, 2003) and so on. Recently, the H_∞ robust control strategy (see Doyle, 2013; Francis, 1987; Zhou K et al., 1996), has been proposed for DP ships (see Katebi, 2011). In (Wang et al., 2012), based on mixed sensitivity, a robust controller is designed for the DP ships with uncertain model. In (Ngongi et al., 2015), the fuzzy controller for DP ships is designed using optimal H_∞ control techniques based on T-S fuzzy model (see Wang et al., 2017a,b); In (You et al., 2017), based on the mixed H_∞ and μ-synthesis framework, the robust control problem for DP ships is discussed.

In reality, the electromagnetic interference, zero shift and actuators aging exist in the DP ship system, so the actuator failures are unavoidable. Therefore, designing a controller to tolerate actuator faults is necessary and important. Recently fault-tolerant control (FTC) has received considerable attentions for DP ships. In (Benetazzo et al.,2015), by combining the Luenberger observer with the parity space approach, a reconfigurable discrete-time variable-structure controller is provided for DP ship with actuators faults. In (Fang et al.,2015), by using the structural reliability-based approach, the two faults of buoyancy element loss and line breakage for DP ship are dealt with efficiently. In (Cristofaro et al.,2014), in order to detect and isolate the actuator and faults, the FTC allocation is presented for overactuated DP ship system by using unknown input observers. In (Su et al., 2016), an FTC controller is developed for DP ship to avoid the actuator saturation and partial loss of actuator effectiveness fault. In (Lin et al.,2016), by using an iterative learning observer, a FTC scheme is developed for DP ships under the unknown environment disturbance and thruster fault. In (Jing et al.,2017), by using a hyper-bolic tangent function, an active disturbance rejection controller is proposed for depth-pitch control of a DP ship.

It is noted that the afore mentioned results about the DP ship system are mainly on continuous-time system. With the development of digital computers, considerable research methods have been adopted for analyzing the sampled-data systems (see Fridman, 2006; Wang et al., 2013; Rubagotti et al., 2011; Lee, 2012; Wu, 2013; Chen et al., 2014; Delchev et al.,2014; Abedi, 2015; Chen, 2015; Wang et al., 2016a,b; Moarref, 2016; Liaquat, 2016; Wang et al.,2017; Yucel et al.,2017). In that system, digital computers are widely used for controlling the continuous-time systems. For example, in DP ship system, a variety of sensors, such as DGPS, gyrocompasses, VRUs, are used by digital computers for determining the ship's position, heading, roll pitch respectively. The sensors produce continuous-time signals, and then the signals are transformed into discrete-time control signals by being sampled and quantized with a microcontroller, and they will be transformed back into continuous-time signals again by the zero-order holder.

Hence, both discrete-time and continuous-time signals exist in one system with a continuous-time framework. So, the sampled-data control systems are more practical and important than the continuous-time systems. Up to now, few literatures have been found to study the problem of sampled-data control for DP ships system. In (Katayama et al.,2010), the nonlinear sampled-data stabilization control for DP ships was studied. In (Hitoshi et al.,2014), the control problem of straight-line trajectory tracking for sampled-data under-actuated ship was discussed. However, almost all the existing literatures have not found to consider the fault-tolerant control for DP ships based on sampled-data, and it has significance both in practice and theory, which motivated this paper.

In this paper, the issue about robust H_∞ fault-tolerant control for sampled-data DP ships with actuator fault is discussed. By input delay approach, the DP ships model is converted to a time-delay system. The major target is to design a H_∞ fault-tolerant sampled-data controller to determine the system's asymptotical stability and achieve H_∞ tracking performance when actuators faults happen. Adequate condition is obtained for the desired controller. Simulation results of a DP ship are provided to show that the system can track the reference signal without steady-state error under the actuator faults and external disturbance like waves, wind, ocean currents.

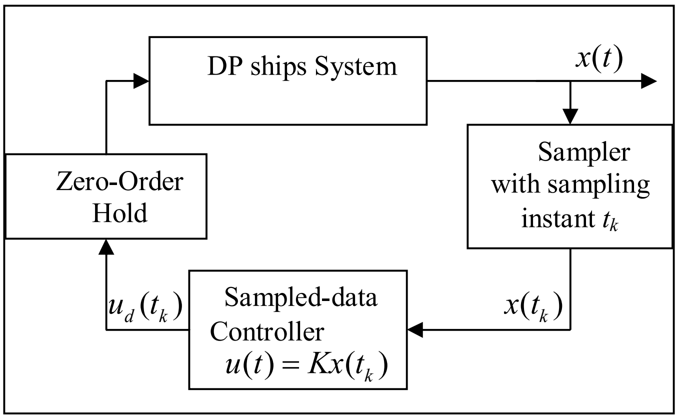

Fig. 1. DP ships control system

2. PROBLEM FORMULATION

The low-speed motion model of DP ship is described as follow:

$$
\begin{aligned}
&M\dot{v}+Dv=u+w\\
&\dot{\eta}=J(\psi)v
\end{aligned}
\tag{1}
$$

Where $\eta=[x\quad y\quad \psi]^T$ represents the ship's position x, position y and heading ψ related to the earth-fixed frame. $\upsilon=[p\quad v\quad r]^T$ represents the ship's velocity of the body-fixed frame, where p represents surge velocity, v represents the sway velocity and r represents the yaw velocity (the body-fixed coordinate system is depicted in Fig.1). u represents the vector of control force and moment; w represents the system's disturbance input. $J(\psi)$ is the transformation matrix of the two frames, and it is defined as

$$
J(\psi)=\begin{bmatrix}\cos(\psi) & -\sin(\psi) & 0\\ \sin(\psi) & \cos(\psi) & 0\\ 0 & 0 & 1\end{bmatrix},
$$

M is the inertia matrix and D is a linear damping matrix, and they are defined as

$$
M=\begin{bmatrix}m-X_{\dot{u}} & 0 & 0\\ 0 & m-Y_{\dot{v}} & mx_G-Y_{\dot{r}}\\ 0 & mx_G-Y_{\dot{r}} & I_z-N_{\dot{r}}\end{bmatrix},
$$

$$
D=\begin{bmatrix}-X_{\dot{u}} & 0 & 0\\ 0 & -Y_{\dot{v}} & -Y_{\dot{r}}\\ 0 & -Y_{\dot{r}} & -N_{\dot{r}}\end{bmatrix}.
$$

Where m represents the mass of ship; I_z represents the inertia moment. $X_{\dot{u}}, Y_{\dot{v}}, Y_{\dot{r}}, N_{\dot{r}}$ are the accessional mass.

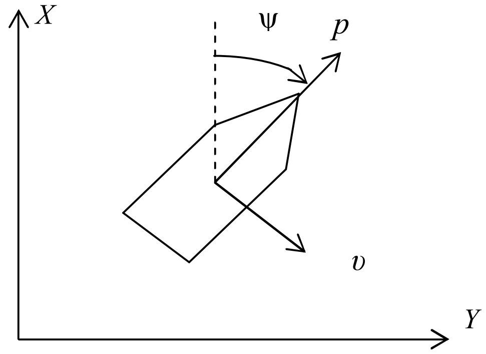

Fig. 2. Coordinate systems of DP ships

As the heading ψ is very small, $J(\psi)$ can be rewritten

$$
J(\psi)\cong I_{3\times 3}. \tag{2}
$$

Under the condition (3), the linearized low frequency motion of the model is got as follows:

$$
\begin{aligned}
&\dot{x}(t)=Ax(t)+Bu(t)+Ew(t)\\
&y(t)=Cx(t)
\end{aligned}
\tag{3}
$$

where

$$
x(t)=[\eta\quad \upsilon]^T=[x\quad y\quad \psi\quad p\quad v\quad r]^T,
$$

$$
A=\begin{bmatrix}0 & I\\ 0 & -M^{-1}D\end{bmatrix},\quad B=\begin{bmatrix}0\\ M^{-1}\end{bmatrix},\quad E=\begin{bmatrix}0\\ M^{-1}\end{bmatrix},
$$

$$
C=\begin{bmatrix}I & 0\end{bmatrix}
$$

Where $y(t)$ represents the control system output, $u(t)$ represents the control input, $w(t)$ represents the exogenous disturbance, C are constant matrixes assumed to be known.

The control object is to consider the tracking control problem when actuator faults exist in the DP ships. This paper considers the loss of actuator effectiveness as the type of fault. Then the following actuator fault model is adopted as follow.

$$u^f(t) = \theta u(t), \quad \theta \in \left[\underline{\theta}, \ \bar{\theta}\right], \quad 0 < \underline{\theta} \le 1, \quad \bar{\theta} \ge 1 \tag{4}$$

where θ represent the uncertain constant, $\underline{\theta}$, $\bar{\theta}$ are the lower and upper bound of θ respectively. When $\underline{\theta}=\bar{\theta}=1$, it means the actuator $u^f(t)$ has no fault.

Define the tracking error as follow

$$e(t) = r(t) - y(t). \tag{5}$$

Where $r(t)$ represents the reference signal. Noted that a controller's tracking error integral plays a role key in eliminating the steady-state tracking error. Similar to [20], letting $z(t) = \int_0^t e(t)dt$, hence, an augmented state-space description for the DP ships system with actuator faults (4) is considered as follow

$$\begin{aligned} \dot{\varsigma}(t) &= \bar{A}\varsigma(t) + \bar{B}\theta u(t) + \bar{E}\bar{w}(t) \\ z(t) &= \bar{C}\varsigma(t) \end{aligned} \tag{6}$$

Where

$$\varsigma(t) = \left[x^T(t) \quad z^T(t)\right]^T, \bar{w}(t) = \left[w^T(t) \quad r^T(t)\right]^T$$

$$\bar{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \bar{E} = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} 0 & I \end{bmatrix}$$

The paper's object is to design a sampling controller such that:

The closed-loop system (6) is asymptotically stable in the event of actuator failures, and the output $y(t)$ tracks the reference signal $r(t)$ without steady-state error;

To reject the external disturbances like waves, wind, ocean currents, the closed-loop system is assumed to satisfy that $\| z(t) \|_2 \le \gamma \| w(t) \|_2$ for all nonzero $w(t) \in L_2\left[0, \infty\right)$ under zero condition, where $\gamma > 0$;

In this paper, the state variables of DP ships are assumed to be measured at the sampling instant $0 = t_0 < t_1 < t_2 < \ldots < t_k < \ldots$, that is, in the interval $t_k \le t \le t_k+1$, only $x(t_k)$ is available. The sampling period follow the assumption that it is bounded by a constant d, that is,

$$t_{k+1} - t_k \le d, \forall k \ge 0, d > 0, \tag{7}$$

Considered the state-feedback control law as follow,

$$u(t) = u_d(t_k) = K\varsigma(t_k) = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x(t_k) \\ \int_0^{t_k} e(t)dt \end{bmatrix}, \quad t_k \le t < t_{k+1}, \tag{8}$$

where t_k represents the sampling instant, $u_d(t_k)$ represents the discrete-time control signal, K is a state-feedback gain matrix. By substituting (8) into (1), then

$$\begin{aligned} \dot{\varsigma}(t) &= \bar{A}\varsigma(t) + \bar{B}\theta K\varsigma(t_k) + \bar{E}\bar{w}(t) \\ z(t) &= \bar{C}\varsigma(t) \end{aligned} \tag{9}$$

Remark 1: Note that both discrete and continuous signals exist in the system (9), which is different from reference (Li et al.,2017) whose control signals are continuous, and it is more difficult to analyse. Besides, compared to the existing continuous-time control method for DP ships system, the sampled-data control method in the paper is different and has more practical significance.

By input delay approach, the system (9) can be converted to the system as follow:

$$\begin{aligned} \dot{\varsigma}(t) &= \bar{A}\varsigma(t) + \bar{B}\theta K\varsigma(t - \tau(t)) + \bar{E}\bar{w}(t) \\ z(t) &= \bar{C}\varsigma(t) \end{aligned} \tag{10}$$

where

$$\tau(t) = t - t_k, \quad t_k \le t < t_{k+1}, \tag{11}$$

$\tau(t)$ is a time-varying delay and satisfies

$$0 \le \tau(t) \le d, \quad \dot{\tau}(t) = 1, \quad t \ne t_k \tag{12}$$

Remark 2: Input delay approach, which is introduced by Fridman et al. (Fridman et al., 2004), is one of the major approaches to analyze the sampled-data system(see Fridman et al., 2004; Wang, 2016; Liu, 2015; Krishnasamy, 2015; Chen, 2015; Wu, 2014),. Its important advantage is that the sampling distance is not needed to be constant. Besides, it can be used to the system with uncertain system parameter (see Gao et al., 2010), which is difficult for traditional lifting techniques to deal with.

3. MAIN RESULTS

The robust fault-tolerant control problem for sampled-data DP ships is studied in this section. By constructing Lyapunov-Krasovskii functional, asymptotical stability condition is derived for the system and the H∞ tracking performance is achieved. Then, the robust fault-tolerant sampled-data controller is obtained by analyzing the stabilization condition.

Theorem 1: Given scale $d > 0$, $\gamma > 0$, the closed-loop system (10) is asymptotically stable with H_∞ tracking performance

γ, if there exist symmetric positive-definite matrices Z, P, Q, M_i, N_i, i=1,2,3,4 such that

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \sqrt{d}M_1 & \sqrt{d}N_1 & \sqrt{d}\bar{A}^T \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \sqrt{d}M_2 & \sqrt{d}N_2 & \sqrt{d}\theta\bar{K}^T\bar{B}^T \\ * & * & \Omega_{33} & -N_4^T & \sqrt{d}M_3 & \sqrt{d}N_3 & 0 \\ * & * & * & -\gamma^2 I & \sqrt{d}M_4 & \sqrt{d}N_4 & \sqrt{d}\bar{E}^T \\ * & * & * & * & -Z & 0 & 0 \\ * & * & * & * & * & -Z & 0 \\ * & * & * & * & * & * & -Z^{-1} \end{bmatrix} < 0, \tag{13}$$

Where

$$\Omega_{11} = P\bar{A} + \bar{A}^T P + Q + M_1 + M_1^T + \bar{C}^T\bar{C},$$
$$\Omega_{12} = P\bar{B}K - M_1 + M_2^T + N_1$$
$$\Omega_{13} = M_3^T - N_1$$
$$\Omega_{14} = P\bar{E} + M_4^T$$
$$\Omega_{22} = -M_2 - M_2^T + N_2 + N_2^T$$
$$\Omega_{23} = -M_3^T - N_2 + N_3^T$$
$$\Omega_{24} = -M_4^T + N_4^T$$
$$\Omega_{33} = -Q - N_3 + N_3^T$$

Proof. First, considered the following Lyapunov-Krasovskii functional

$$V(t) = \sum_{i=1}^{3} V_i(t), \quad t \in [t_k, t_{k+1}) \tag{14}$$

$$V_1(t) = \varsigma(t)^T P\varsigma(t) \tag{15}$$

$$V_2(t) = \int_{t-d}^{t} \varsigma(s)^T Q\varsigma(s)ds \tag{16}$$

$$V_3(t) = \int_{-d}^{0}\int_{t+\theta}^{t} \dot{\varsigma}(s)^T Z\dot{\varsigma}(s)dsd\theta \tag{17}$$

Calculating the derivative of $V(t)$, it can be obtained that:

$$\begin{aligned} \dot{V}_1(t) &= 2\varsigma(t)^T P\dot{\varsigma}(t) \\ \dot{V}_2(t) &= \varsigma(t)^T Q\varsigma(t) - \varsigma(t-d)^T Q\varsigma(t-d) \\ \dot{V}_3(t) &= d\dot{\varsigma}(t)^T Z\dot{\varsigma}(t) - \int_{t-d}^{t} \dot{\varsigma}(s)^T Z\dot{\varsigma}(s)ds \end{aligned} \tag{18}$$

Using the Leibniz-Newton formula, for any matrices of proper dimension $M_i, N_i,\ i = 1,2,3,4$, the equations can be obtained as follow:

$$2[\varsigma^T(t)M_1 + \varsigma^T(t-\tau(t))M_2 + \varsigma^T(t-d)M_3 - \bar{w}^T(t)M_4] \times[\varsigma(t) - \varsigma(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{\varsigma}(s)ds] = 0 \tag{19}$$

$$2[\varsigma^T(t)N_1 + \varsigma^T(t-\tau(t))N_2 + \varsigma^T(t-d)N_3 + \bar{w}^T(t)N_4] \times[\varsigma(t-\tau(t)) - \varsigma(t-d) - \int_{t-d}^{t-\tau(t)} \dot{\varsigma}(s)ds] = 0 \tag{20}$$

Similarly, it can be obtained that

$$\int_{t-d}^{t} \dot{\varsigma}(s)Z\dot{\varsigma}(s)ds = \int_{t-\tau(t)}^{t} \dot{\varsigma}(s)Z\dot{\varsigma}(s)ds + \int_{t-d}^{t-\tau(t)} \dot{\varsigma}(s)Z\dot{\varsigma}(s)ds \tag{21}$$

Substituting (19)-(21) into (18) that

$$\begin{aligned} \dot{V}(t) \le \zeta^T(t)(\Theta + d&\begin{bmatrix}\bar{A} & \theta\bar{B}K & 0 & \bar{E}\end{bmatrix}^T \\ &\cdot Z\begin{bmatrix}\bar{A} & \theta\bar{B}K & 0 & \bar{E}\end{bmatrix} \\ &+ dMZ^{-1}M^T + dNZ^{-1}N^T)\zeta(t) \\ &- \int_{t-\tau(t)}^{t}[\zeta^T(t)M + \dot{\varsigma}^T(s)Z]Z^{-1}[M^T\zeta(t) + Z\dot{\varsigma}(s)]ds \\ &- \int_{t-d}^{t-\tau(t)}[\zeta^T(t)N + \dot{\varsigma}^T(s)Z]Z^{-1}[N^T\zeta(t) + Z\dot{\varsigma}(s)]ds \end{aligned} \tag{22}$$

where

$$\zeta(t) = \begin{bmatrix}\varsigma^T(t) & \varsigma^T(t-\tau(t)) & \varsigma^T(t-d) & \bar{w}(t)\end{bmatrix}^T$$
$$M = \begin{bmatrix}M_1^T & M_2^T & M_3^T & M_4^T\end{bmatrix}^T,$$
$$N = \begin{bmatrix}N_1^T & N_2^T & N_3^T & N_4^T\end{bmatrix}^T,$$

$$\Theta = \begin{bmatrix} \Omega_{11} & \Omega_{12} & M_3^T - N_1 & PB + M_4^T \\ * & \Omega_{22} & -M_3^T - N_2 + N_3^T & M_4^T + N_4^T \\ * & * & -Q - N_3 - N_3^T & -N_4^T \\ * & * & * & -\gamma^2 I \end{bmatrix} \tag{23}$$

Since Z>0, then the last two parts in (22) are all less than 0. According to Schur complement, (13) implies that

$$\Theta + d\begin{bmatrix}\bar{A} & \theta\bar{B}K & 0 & \bar{E}\end{bmatrix}^T Z\begin{bmatrix}\bar{A} & \theta\bar{B}K & 0 & \bar{E}\end{bmatrix} + dMZ^{-1}M + dNZ^{-1}N < 0 \tag{24}$$

Thus, from Eq. (24), it can be obtained that

$$z^T(t)z(t) - \gamma^2\bar{w}^T(t)\bar{w}(t) + \dot{V}(t) < 0 \tag{25}$$

Under zero conditions, there are $V(0)=0$ and $V(\infty)\geq 0$. From Eq. (25), $\|z(t)\|_2 \leq \gamma \|w(t)\|_2$ can be got for all nonzero $w(t)\in L_2[0,\infty)$, then the H_∞ tracking performance is established. This proof is completed.

Remark 3. In order to achieve the H_∞ tracking performance for system(10), free-weighting matrix approach combined with Lyapunov-Krasovskii functional are used in Theorem 1. It is noted that free-weighting matrix approach play a role key in reducing conservativeness of proposed delay-dependent results.

Now, based on Theorem 2, the robust fault-tolerant sampled-data controller (8) will be designed.

Theorem 2: Given scales $d>0$, $\gamma>0$, the system (10) is asymptotically stable under the assumption that the sampling period is bounded by a constant d, that is, $t_{k+1}-t_k\leq d, \forall k\geq 0$,if there exist symmetric positive-definite matrices $\tilde{P}>0, \tilde{Q}>0, \tilde{Z}>0$, $\tilde{M}_j, \tilde{N}_j, j=1,2,3,4$, such that the LMIs hold as follow :

$$\begin{bmatrix} \tilde{\Omega}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} & \tilde{\Omega}_{14} & \sqrt{d}\tilde{M}_1 & \sqrt{d}\tilde{N}_1 & \sqrt{d}\tilde{P}\bar{A}^T & \tilde{P}\bar{C}^T \\ * & \tilde{\Omega}_{22} & \tilde{\Omega}_{23} & \tilde{\Omega}_{24} & \sqrt{d}\tilde{M}_2 & \sqrt{d}\tilde{N}_2 & \sqrt{d}\theta\tilde{K}^T\bar{B}^T & 0 \\ * & * & \tilde{\Omega}_{33} & -\tilde{N}_4^T & \sqrt{d}\tilde{M}_3 & \sqrt{d}\tilde{N}_3 & 0 & 0 \\ * & * & * & -\gamma^2 I & \sqrt{d}\tilde{M}_4 & \sqrt{d}\tilde{N}_4 & \sqrt{d}\tilde{P}\bar{E}^T & 0 \\ * & * & * & * & -\tilde{Z} & 0 & 0 & 0 \\ * & * & * & * & * & -\tilde{Z} & 0 & 0 \\ * & * & * & * & * & * & \tilde{Z}-2\tilde{P} & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{26}$$

where

$$\tilde{\Omega}_{11} = \tilde{P}\bar{A} + \bar{A}^T\tilde{P} + \tilde{Q} + \tilde{M}_1 + \tilde{M}_1^T$$
$$\tilde{\Omega}_{12} = \bar{B}\tilde{K} - \tilde{M}_1 + \tilde{M}_2^T + \tilde{N}_1$$
$$\tilde{\Omega}_{13} = \tilde{M}_3^T - \tilde{N}_1$$
$$\tilde{\Omega}_{14} = \bar{E} + \tilde{M}_4^T$$
$$\tilde{\Omega}_{22} = -\tilde{M}_2 - \tilde{M}_2^T + \tilde{N}_2 + \tilde{N}_2^T$$
$$\tilde{\Omega}_{23} = -\tilde{M}_3^T - \tilde{N}_2 + \tilde{N}_3^T$$
$$\tilde{\Omega}_{24} = -\tilde{M}_4^T + \tilde{N}_4^T$$
$$\tilde{\Omega}_{33} = -\tilde{Q} - \tilde{N}_3 + \tilde{N}_3^T$$

Moreover, a suitable controller with H_∞ performance γ in forms of (8) is designed to satisfy the proposed conditions. And the control gain matrix K is obtained as

$$K = \begin{bmatrix} K_1 & K_2 \end{bmatrix} = \tilde{K}\tilde{P}^{-1} \tag{27}$$

Proof: Noting that $\tilde{Z}>0$, it can be obtained that

$$(\tilde{Z}-\tilde{P})\tilde{Z}^{-1}(\tilde{Z}-\tilde{P})>0, \tag{28}$$

which is equivalent to

$$-\tilde{P}\tilde{Z}^{-1}\tilde{P} \leq \tilde{Z} - 2\tilde{P} \tag{29}$$

Let $\eta = diag\{P^{-T}, P^{-T}, P^{-T}, I, P^{-T}, P^{-T}, I, I\}$.
Denoting

$$\tilde{P}=P^{-1}, \tilde{K}=KP^{-1}, \tilde{Q}=P^{-T}QP^{-1},$$
$$\tilde{M}_1 = P^{-T}M_1P^{-1}, \tilde{M}_4 = M_4P^{-1}, \tilde{Z} = P^{-T}ZP^{-1}$$

and pre- and post-multiplying Eq. (13) by η and η^T respectively, the following inequality can be obtained:

$$\begin{bmatrix} \tilde{\Pi}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} & \tilde{\Omega}_{14} & \sqrt{d}\tilde{M}_1 & \sqrt{d}\tilde{N}_1 & \sqrt{d}\tilde{P}\bar{A}^T \\ * & \tilde{\Omega}_{22} & \tilde{\Omega}_{23} & \tilde{\Omega}_{24} & \sqrt{d}\tilde{M}_2 & \sqrt{d}\tilde{N}_2 & \sqrt{d}\theta\tilde{K}^T\bar{B}^T \\ * & * & \tilde{\Omega}_{33} & -\tilde{N}_4^T & \sqrt{d}\tilde{M}_3 & \sqrt{d}\tilde{N}_3 & 0 \\ * & * & * & -\gamma^2 I & \sqrt{d}\tilde{M}_4 & \sqrt{d}\tilde{N}_4 & \sqrt{d}\tilde{P}\bar{E}^T \\ * & * & * & * & -\tilde{Z} & 0 & 0 \\ * & * & * & * & * & -\tilde{Z} & 0 \\ * & * & * & * & * & * & -\tilde{Z}^{-1} \end{bmatrix} < 0, \tag{30}$$

where

$$\tilde{\Pi}_{11} = \tilde{P}\bar{A} + \bar{A}^T\tilde{P} + \tilde{Q} + \tilde{M}_1 + \tilde{M}_1^T + \tilde{P}C^TC\tilde{P}$$

by Schur complement, Eq. (26) can be obtained. This proof is completed.

4. NUMERICAL EXAMPLES

For validating the effectiveness of the proposed methods, a numerical simulation is carried out for a DP ship with environment disturbance in the section. The length of vessel is 76.2m, tonnage is 4.591×106 kg. Similar with (Tannuri et al.,2010), the M and D in model (1) are given by:

$$M = \begin{bmatrix} 5.3122\times10^6 & 0 & 0 \\ 0 & 8.2831\times10^6 & 0 \\ 0 & 0 & 3.7454\times10^9 \end{bmatrix}$$

$$D = \begin{bmatrix} 5.0242\times10^4 & 0 & 0 \\ 0 & 2.7229\times10^5 & -4.3933\times10^6 \\ 0 & -4.3933\times10^6 & 4.1894\times10^8 \end{bmatrix}$$

Similar to (Li et al.,2017), the environment disturbances w(t) is considered as

$$w = J^T(\psi)b \tag{31}$$

where $J(\psi)$ is defined in (1), and $b \in \Re^3$ is a vector of bias forces and moment. For simulation, this paper assumed that $b = \begin{bmatrix} 0.3 & 0.5 & -0.1 \end{bmatrix}$, the ship's initial states $x_s(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. In addition, considering the actuator fault, to set θ=0.3. Then, when the sampling interval d=1s, the minimum value achieved by Theorem 2 to guarantee H_∞ performance γ_{min}=1.3652. And the value of feedback gain matrix is obtained that

$$K_1 = \begin{bmatrix} -0.0193 & 0 & 0 & -0.0570 & 0 & 0 \\ 0 & -0.0331 & -0.0036 & -0.0002 & -0.0943 & 0.0091 \\ 0.0037 & 3.5501 & -9.1053 & 0.1109 & 4.1680 & -33.1740 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 0 \\ -0.0036 \\ -9.1052 \end{bmatrix}$$

The fault-tolerant sampled-data controllers is applied to the system (10). When $r(t) = 0.5$ and $r(t) = 0.5\sin 4t$, the ship's output y(t) and the reference signal r(t) are shown in Fig. 3 and 5 respectively, and the error between y(t) and r(t) are shown in Fig. 4 and 6 respectively, from which it can be seen that whether r(t) is a constant or a variable, when there exist actuator fault (at about t=0.3s in this case) and external disturbances, y(t) can still track r(t) well. This indicates that the DP ships has an acceptable tracking performance when the external disturbances and actuator fault happens.

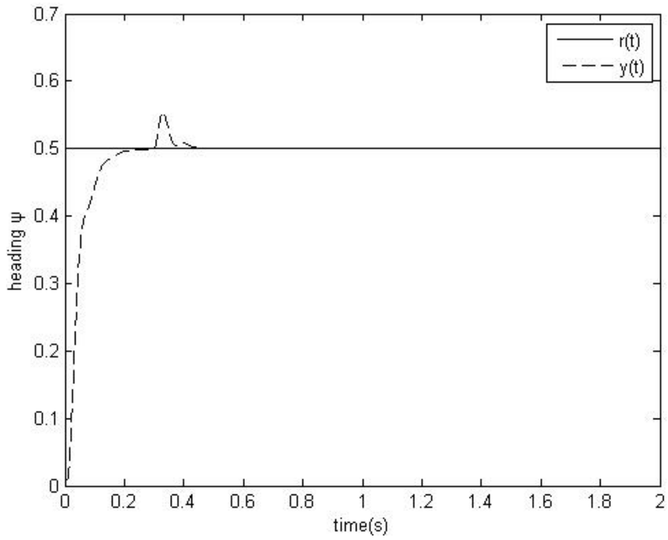

Fig.3 the output $y(t)$ and the reference signal r(t) when $r(t)$=0.5

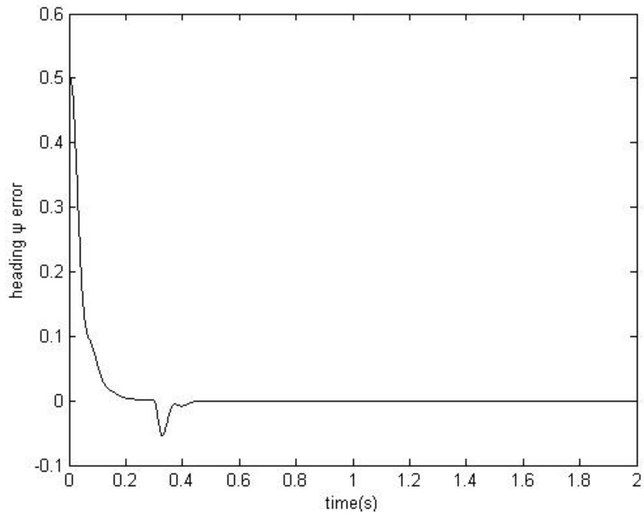

Fig.4 the error between $y(t)$ and $r(t)$ when $r(t)$=0.5

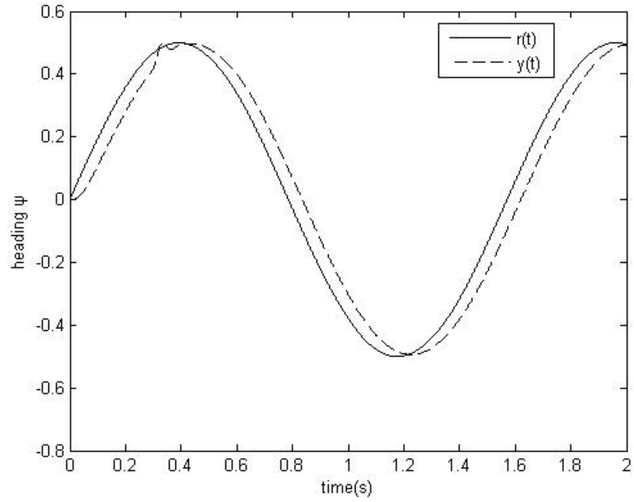

Fig.5 the output $y(t)$ and the reference signal $r(t)$ when $r(t)$=0.5sin4t

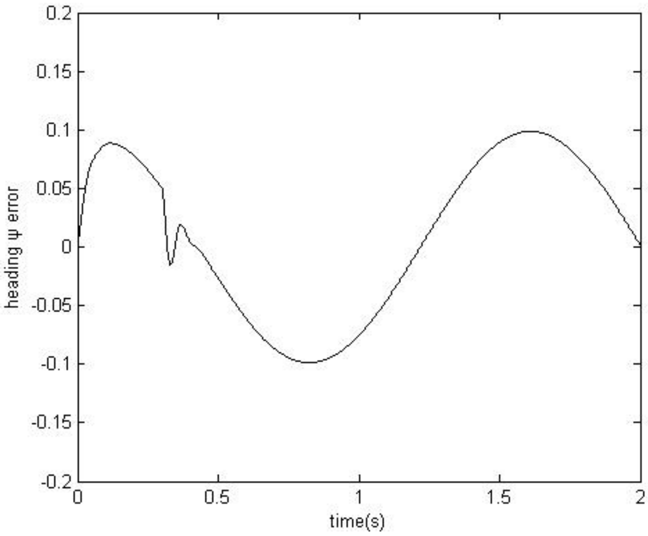

Fig.6 the error between $y(t)$ and $r(t)$ when r(t)=0.5sin4t

5. CONCLUSIONS

A robust H_∞ fault-tolerant control problem for DP ships based on sampled-data with actuator fault is discussed in this paper. The DP ships linearized model with sampled-data is converted to a time-delay system by input delay approach. Then a H_∞ fault-tolerant sampled-data controller is designed to determine the system's asymptotical stability and achieve H_∞ tracking performance. Simulation results of a DP ship are provided to show the effectiveness of the obtained method. And the proposed methods can be applied to other ships.

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