CEAI, Vol.23, No.1, pp. 103-114, 2021 Printed in Romania

Grey Wolf Optimization Algorithm Based Weight Selection for Tuning H-Infinity Loop Shaping Controller in Application to A Benchmark Multivariable System with Transmission Zero

J. Indirapriyadharshini*, S. Manoharan**

**Department of MCT, Sri Krishna College of Engineering and Technology, Coimbatore-641008, India(Tel:91-8754041166,e-mail:indiravasanthraj@gmail.com)*
***Department of EIE, Karpagam College of Engineering, Coimbatore-641032, India (e-mail: manoish07@gmail.com)*

Abstract: This paper presents a method using the Grey-Wolf Optimization (GWO) algorithm for H-Infinity Loop Shaping Controller (HILSC) design applicable to the multivariable system with non-minimum phase and minimum phase. The role of GWO is to determine the weighing function parameters of the HILSC by minimizing the norm (H-infinity) of the Closed-Loop Transfer Function (CLTF) of the multivariable process which is a quadruple tank system. The optimal stability margin in a HILSC is taken as an objective function of GWO to compute the utmost appropriate weighting parameters. The mathematical equivalent for the non-minimum phase and minimum phase of the quadruple tank system is obtained in state-space form by moving the operating position of the valve and the benchmark HILS procedure is employed to determine the weighting functions of the HILSC. The difficulties usually arise in finding compensators that implement most design decisions. It is revealed that the proposed method subsequently reduces the technical hitches in the designing method of the H-infinity controller for a Multivariable System (MVS). GWO based optimization provides comparatively better results than GA and PSO based optimization for a quadruple tank system, which is evident in the simulation results obtained against plant perturbations and disturbances. The integral square error is reduced appreciably with the proposed technique. The necessary computations and the analysis of the behavior of the system are done in a MATLAB/ Simulink environment.

Keywords: Grey Wolf Optimization (GWO), H-Infinity Loop Shaping Controller (HILSC), Multivariable control system, HILS (H-infinity loop shaping), Closed-Loop Transfer Function (CLTF), Non-minimum phase systems, Multivariable System (MVS), Genetic Algorithm (GA), Particle Swarm Optimization (PSO).

1. INTRODUCTION

Most of the complicated engineering structures are developed with many actuators which can influence their behavior (dynamic and static as well). In cases where few shapes of automatic (computerized) control are required over the system, numerous sensors are employed to reveal the information of process variable(s) measurement, which are used for feedback control of the system. A system is said to be an MVS (Multivariable System) or a MIMO (Multi-Input-Multi-Output) system when it possesses multiple actuating input control/sensor output. The critical goal for an MVS is manipulating the input variables and input from channels of the system, simultaneously to obtain the desired output.

Many control problems in process industries are nonlinear and feature more than one controlled variable which can be unusual properties for mathematically derived models of extensive uncertainties, robust interactions, and non-minimum phase behavior. So it loads important for control engineer, chemical engineer to recognize the non-idealities of industrial systems through sporting out experiments with a piece of good laboratory equipment (Shneiderman et al., 2010).

The quadruple tank's general mathematical model can retain all the properties of the system, but the location of the zeros can be determined by the values of $\gamma 1$ and $\gamma 2$ that are set (Corriou Jean-Pierre, 2018). The physical system parameters that are determined through mathematical modeling cannot be accurate. The major issues with mathematical modeling are inaccurate model parameters which may occur due to various factors. The parameter values are subjected to change with the change in time and different characteristics. The uncertainty exists between the actual system and mathematically modeled system because of their difference with test results (Ho et al., 2005; McFarlane et al., 1992).

To deal with an uncertain plant, an appropriate uncertainty model is to be decided on and the complete system dynamics are also to be taken into consideration. Robust control provides essential techniques based totally upon an incomplete description of the controlled process applicable within the regions of non-linear and time-various approaches, which includes MIMO (Multi-Input and Multi-Output) dynamic systems.

Glover and McFarlane proposed a HILS design method

(McFarlane et al., 1992) that consists of open-loop shaping through a pair of compensators to achieve overall performance/stability trade-offs. Designers are to follow the design procedure to develop their system based on the open-loop characteristics for determining the compensators for loop shaping and a suitable weighting function selection that results in a robust controller. Although this approach is properly-organized to design a structure specified robust controller; however, the choice of uncertainty weight of their methods is complex. Hereto, in the MIMO system, the issue of uncertainty in weight selection will become an essential venture to be addressed.

To conquer this drawback of H-infinity optimal control, McFarlane, and Glover (McFarlane et al., 1992) proposed an approach referred to as H-infinity loop shaping control. This approach was primarily built with the idea of the open-loop shaping of a system by simply using two compensator weights that was to be decided on. Fortunately, the compensator weight selection way on this technique was very apparent via the classical control theorem. Therefore, there is still a lack of concrete ideas on how to define the weighting characteristics (ShenhuaYang and Minjie Zheng, 2018). One simple motive is that the LSDP itself can deal with a huge variety of control complications and issues based on the systematic selection of weighing functions. This has motivated to find such a specific method by restricting a class of goal in control systems.

In this proposed work, GWO is implemented to select optimal weighting functions of the structure-specified *H*-infinity loop shaping controller of a multivariable system (Quadruple tank system in this case) for minimum and non- minimum-phase modes. The quadruple tank process model is initially formed by the pre-compensator (W_1) and the post-compensator (W_2) to achieve the desired open-loop form and to determine the structure of the actual robust controller. Eventually, GWO is used to check for the parameters of the design controller in a way that the stability margin is reduced. The outcomes of the proposed method have been finally compared with the conventional *H*-infinity loop shaping approach, GA and PSO algorithms.

2. OPERATION AND MODELING OF QUADRUPLE TANK SYSTEM

The quadruple tank brought by Johansson (Johansson, 2000) gained first-rate publicity as it exhibits fascinating characteristics in both teaching and research. The quadruple tank displays complex dynamics elegantly and in reality. The dynamic characteristics include interactions and a zero-place transmission that can be modified in operation. With better tuning, this system provides non-minimum phase characteristics that arise because of the MIMO nature of the system. And so, the quadruple tank was used to elucidate the effects of different control techniques and as a training device in teaching superior MIMO system characteristics and their control methods. (Salim and Khosrowjerdi,2017).

The four tanks are connected with two pumps and to form the structure of the quadruple tank process which is presented in Figure 1. The parameters v_1 and v_2 (enter voltages to the pumps) are considered as process inputs and parameters y_1 and y_2 (voltages from level measuring devices) are considered as process outputs. The quadruple tank system is designed such that the desired level of lower tanks is achieved by inlet flow rate variation.

Therefore, each pump flow goes to 2 tanks, one decrease and any other upper one, diagonally opposite, and the break-up ratio is controlled by the three-way valve position. With any adjustment in the location of the 2 valves, the mechanism can be either within the minimum phase or within the non-minimal phase, as it should be.

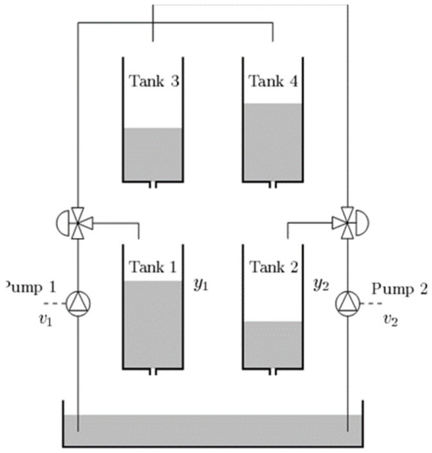


Fig. 1. Quadruple Tank Process.

Let the parameter γ (inlet flow rate variation parameter) be calculated by the setting of the valves. When γ_1 is the flow ratio to the first tank, the flow to the fourth tank is (1-γ_1). Likewise, if γ_2 is the flow ratio to the second tank, then the flow to the third tank is (1-γ_2). The voltage added to the '*i*' pump is v_i, and $k_i v_i$ is the equivalent flow rate.

Based on how the valves are placed before an experiment, the parameters γ_1, γ_2 ε (0, 1) are calculated. The flow to the tank 1 is $\gamma_1 k_c v_1$ and the flow to the tank 4 is (1-γ_1) $k_c v_1$ and similarly to the tank 2 and tank 3. Gravity force is referred to as' g.' The determined signal rates $y_1 = k_c h_1$ and $y_2 = k_c h_2$, where k_c is the co-efficient of pump discharge.

Equations (1)-(4) represent the quadruple tank system in non-linear form as

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_c}{A_1} v_1 \quad (1)$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_c}{A_2} v_2 \quad (2)$$

$$\frac{dh_3}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_c}{A_2} v_2 \quad (3)$$

$$\frac{dh_4}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_c}{A_2} v_2 \quad (4)$$

where

A_i is the cross-sectional area of Tank 'i'
a_i is a cross-section of outlet hole of Tank 'i'
h_i is the water level in Tank 'i'.

The tank levels (h_i) of all four tanks in the quadruple tank cycle are used as the state variables (x_i), the voltages added to pumps (v_1 and v_2) are the input variables (u_i), and the output variables (y_i) are the amounts of tank 1 and tank 2.

For linearization, the following Equations are used, $x_i= h_i-h_i^0$; $u_i= v_i-v_i^0$, where h_i^0 and v_i^0 are the h_i and v_i defined values. The linearized state-space model with function matrices is obtained using the expansion of the Taylor series set out in (5) and the configuration is given in (6) and (7).

$$\dot{x_i} = \dot{h_i} \approx f(h_i^o, v_i^o) + \frac{\partial f(h,v)}{\partial h^T} /_{h_i=h_i^o} \; (h_i - h_i^o) + \frac{\partial f(h,v)}{\partial v^T} /_{v_i=v_i^o} \; (v_i - v_i^o) \quad (5)$$

$$f(h_i^o, v_i^o) \xrightarrow{yields} 0; \frac{\partial f(h,v)}{\partial h^T} \xrightarrow{yields} \boldsymbol{A}; \quad (6)$$

$$\frac{\partial f(h,v)}{\partial h^T} /_{h_i=h_i^o} \xrightarrow{yields} \boldsymbol{X}; \quad (7)$$

$$\frac{\partial f(h,v)}{\partial v^T} /_{v_i=v_i^o} \xrightarrow{yields} \boldsymbol{B}; \quad (8)$$

$$(v_i - v_i^o) \xrightarrow{yields} \boldsymbol{U}; \quad (9)$$

The linearized model is given by

$$X = A\dot{X} + BU$$
$$Y = CX + DU \quad (10)$$

Where,

$$A = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & 0 \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} and\ D = 0; Statevector, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

The time constant is calculated from the equation (11),

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^o}{g}}, i = 1, \dots, 4 \quad (11)$$

The steady-state operating points and process parameter values are assumed as given in the literature (Johansson K H, 2000), and the values are tabulated respectively in Table 1 and Table 2.

Table 1. Process Parameters.

Area of Tank 1 and 3 (A_1, A_3)	28 cm^2
Area of Tank 2 and 4 (A_2, A_4)	32 cm^2
a_1, a_3	0.071 cm^2
a_2, a_4	0.057 cm^2
k_c	0.5 V/cm

Table 2. Steady-state Operating Points.

Steady-state parameters	Minimum Phase	Non minimum Phase
h_1^o, h_2^o [cm]	(12.4, 12 .7)	(12 .6, 13)
h_3^o, h_4^o [cm]	(1 .8, 1 .4)	(4 .8, 4 .9)
v_1^o, v_2^o [V]	(3.00, 3 .00)	(3 .15, 3 .15)
k_1, k_2 [cm^3/V s]	(3.33, 3.35)	(3 .14, 3 .29)
γ_1, γ_2	(0 .70, 0.60)	(0 .43, 0 .34)

The state-space matrices of the quadruple tank system A, B, and C are determined by removing the values above in (11). The transfer function matrices are obtained using the MATLAB function and are given in (12) and (13) respectively for minimum phase and non-minimum phase operating points.

$$G_{\min(s)} = \begin{bmatrix} \frac{2.6}{1+62s} & \frac{1.5}{(1+23s)(1+62s)} \\ \frac{1.4}{(1+30s)(1+90s)} & \frac{2.5}{1+90s} \end{bmatrix} \quad (12)$$

$$G_{\text{non\_min}(s)} = \begin{bmatrix} \frac{1.5}{1+63s} & \frac{2.5}{(1+39s)(1+63s)} \\ \frac{2.5}{(1+56s)(1+91s)} & \frac{2.5}{1+91s} \end{bmatrix} \quad (13)$$

The generated transfer matrices $G_{\min(s)}$ and $G_{\text{non-}\min(s)}$ usually has two zeros in which one of the zeros is always positioned in the left half of the s-plane, while another zero can be put either in the left half or the right half of the s-plane, depending on the location of the three-way valve. As a consequence, the mechanism is said to be in the minimum phase if the valves γ_1 and γ_2 are in the $0 < \gamma_1 + \gamma_2 < 1$and non-minimum phase, if the valves γ_1 and γ_2 are in the position $1 < \gamma_1 + \gamma_2 < 2$.

3. DESIGN OF H-infinity LOOP SHAPING CONTROLLER

For scalar feedback structure, H-infinity loop-shaping is well known and commonly used. For a scalar system, converting closed-loop disturbance rejection requirements into loop-shape requirements is a straightforward process.H-infinity Loop shaping is mainly based on weighting the corresponding inputs and outputs of the nominal plant, where it has several advantages regarding the classical methods (Skogestad et al., 1996).

McFarlane and Glover (McFarlane et al., 1992) suggested an appropriate method for the configuration of the H-infinity loop shaping mechanism, which was a solution to a wide range of control problems. The uncertainties are represented in this approach as co-prime uncertainty and this concept does not reflect actual physical uncertainty. W_1 (pre-compensator) and W_2 (post-compensator) are the two weighting functions that set out to shape the real plant G in order to achieve the desired open-loop structure. The shaped plant G_s obtained by this method is expressed as

$$G_s = W_1 G W_2 = \begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} \quad (14)$$

$$G_s = (N_s + \Delta_{NS})((M_s + \Delta_{MS})^{-1} \quad (15)$$

Where A_s, B_s, C_s, D_s represent shaped plant, Gs in the state-space form,

$$\|\Delta_{NS}, \quad \Delta_{MS}\|_\infty \leq \varepsilon \tag{16}$$

where N_S and M_S are nominators and denominators of generalized coprime variables. Δ_{NS} and Δ_{MS} are ambiguity mapping mechanisms with nominator and denominator variables. ε is an uncertainty boundary, called a stability margin (McFarlane et al., 1992).

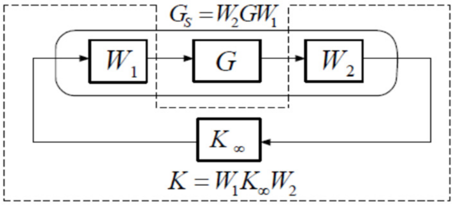


Fig. 2. Block Diagram of H-infinity controller.

The following steps are suggested for the design of a typical H-infinity loop shaping controller based on (McFarlane et al., 1992).

1. The selection of W_1 and W_2 such that the shaped plant Gs has no hidden approaches. The pre-compensator W_1 is usually chosen to monitor output and interference rejection, and W_2 is chosen to reduce sensor noise. Essentially, W_1 is chosen as a weighting function with an essential action to make a zero steady-state error. The post-compensator W_2 can be selected as an identity matrix to remove the noise influence of the system when the right sensor is used.

For minimum phase, the pre- compensator W_1 is calculated as follows:

$$W_{1\_H-infinity\_MP} = K_W \frac{s+a}{s+b} = \frac{2s+0.1}{0.04s+0.002} \tag{17}$$

The pre-compensator W_1 determined for the non-minimum phase is

$$W_{1\_H-infinity\_NMP} = K_W \frac{s+a}{s+b} = \frac{12s+12}{12s+0.015} \tag{18}$$

Where K_w and a and b are positive numbers in which provides integral action with the value chosen as (<<1).

2. The infinity norm is to be minimized for designing overall stabilizing controllers' K and the optimal cost γ_{opt} is calculated. The stability margin ε and ε_{opt} are the measure of robustness of the desired loop shape.

For minimum phase and non-minimum phase, the optimal values are calculated using the formula in equation (19) and (20) as 14.22 and 30

$$\gamma_{opt\_MP} = \varepsilon^{-1}{}_{opt} = inf \left\| \begin{bmatrix} I \\ K \end{bmatrix} (1 + G_S K)^{-1} M_S^{-1} \right\|_\infty = 14.22 \tag{19}$$

For the non-minimum phase:

$$\gamma_{opt\_NMP} = \varepsilon^{-1}{}_{opt} = inf \left\| \begin{bmatrix} I \\ K \end{bmatrix} (1 + G_S K)^{-1} M_S^{-1} \right\|_\infty = 30 \tag{20}$$

3. Choose $\varepsilon<\varepsilon_{opt}$, and evaluate controller K_∞. The state model of the H-infinity controller is given as follows:

For minimum phase:

$$A_{H\_\inf\_MP} = \begin{bmatrix} -0.03743 & -0.006521 & 0.02562 \\ 0.003425 & -0.1896 & -0.01235 \\ -0.02082 & -0.02413 & -0.04583 \end{bmatrix}$$

$$B_{H\_\inf\_MP} = \begin{bmatrix} -0.04783 & -0.1299 \\ 0.3632 & -0.2446 \\ 0.04297 & -0.02071 \end{bmatrix}$$

$$C_{H\_\inf\_MP} = \begin{bmatrix} 0.0633 & 0.3253 & 0.02398 \\ 0..09166 & -0.2598 & -0.01056 \end{bmatrix}$$

$$D_{H\_\inf\_MP} = \begin{bmatrix} -0.7777 & 0.5063 \\ 0.5708 & -0.3716 \end{bmatrix} \tag{21}$$

For non-minimum phase,

$$A_{H\_\inf\_NMP} = \begin{bmatrix} -0.154 & 0.3062 & -0.1839 & -0.3508 & -0.04948 \\ -0.07386 & -2.918 & -0.1102 & 3.282 & 0.1133 \\ 0.07244 & -0.256 & -0.07941 & 0.2927 & 0.04634 \\ -0.04896 & -1.81 & 0.07437 & 1.76 & -0.0605 \\ 0.01868 & -0.1383 & -0.03797 & 0.1408 & -0.05669 \end{bmatrix}$$

$$B_{H\_\inf\_NMP} = \begin{bmatrix} -0.637 & -7.705 \\ 10.47 & -0.6085 \\ 0.9965 & -4.538 \\ 6.185 & -4.538 \\ 0.4919 & -0.3636 \end{bmatrix}$$

$$C_{H\_\inf\_NMP} = \begin{bmatrix} -0.07843 & 5.425 & -0.2746 & -5.85 & -0.2053 \\ -0.3311 & -4.444 & -0.1094 & 4.715 & 0.1858 \end{bmatrix}$$

$$D_{H\_\inf\_NMP} = \begin{bmatrix} -19.29 & 14.16 \\ 15.59 & -11.44 \end{bmatrix} \tag{22}$$

4. Final overall controller (K) follows using loop shaping method is determined using equation (23),

$$K=W_1 K_\infty W_2 \tag{23}$$

The state model of the final overall controllers is designed as follows:

For the Minimum phase:

$$A_{K\_H\_\inf\_MP} = \begin{bmatrix} -0.03743 & 0 & -0.0633 & -0.3253 & -0.0.2398 \\ -0.05 & -0.05 & -0.09166 & 0.2598 & 0.01056 \\ 0 & 0 & -0.03743 & -0.006521 & 0.01056 \\ 0 & 0 & 0.003425 & -0.1896 & -0.01235 \\ 0 & 0 & -0.02082 & -0.02413 & -0.04583 \end{bmatrix}$$

$$B_{K\_H\_\inf\_MP} = \begin{bmatrix} 0.7777 & -0.5063 \\ 0.5708 & 0.3716 \\ -0.04783 & -0.1299 \\ 0.3632 & -0.2446 \\ 0.04297 & -0.02071 \end{bmatrix}$$

$$C_{K\_H\_\inf\_MP} = \begin{bmatrix} 0 & 0 & -0.03165 & -0.1627 & -0.01199 \\ 0 & 0 & -0.04583 & 0.1299 & 0.00528 \end{bmatrix}$$

$$D_{K\_H\_\inf\_MP} = \begin{bmatrix} 0.3888 & -0.2531 \\ -0.2854 & 0.1858 \end{bmatrix} \tag{24}$$

For the non-minimum phase:

$A_{K\_H\_\inf\_NMP}$

$$= \begin{bmatrix} -0.00125 & 0 & 0.07843 & -5.425 & -0.2756 & 5.85 & 0.2053 \\ 0 & -0.00125 & 0.3311 & 4.444 & 0.1094 & -4.715 & -0.1685 \\ 0 & 0 & -0.1534 & 0.3062 & -0.1839 & -0.3508 & 0.04948 \\ 0 & 0 & -0.07386 & -2.918 & -0.1102 & 3.282 & 0.1133 \\ 0 & 0 & 0.07244 & -0.256 & -0.07941 & 0.2927 & 0.04634 \\ 0 & 0 & -0.04896 & -1.81 & -0.07437 & 1.76 & 0.0605 \\ 0 & 0 & 0.01868 & -0.1383 & -0.03797 & 0.1408 & -0.05669 \end{bmatrix}$$

$$B_{K\_H\_\inf\_NMP} = \begin{bmatrix} 19.29 & -14.16 \\ -15.59 & 11.44 \\ -0.637 & -4.538 \\ 10.47 & -4.538 \\ 0.9965 & -0.3636 \\ 6.185 & -4.538 \\ 0.4919 & -0.3636 \end{bmatrix} s$$

$$C_{K\_H\_\inf\_NMP} = \begin{bmatrix} 0.9988 & 0 & 0.07843 & -5.425 & -0.2756 & 5.85 & 0.2053 \\ 0 & 0.9988 & 0.3311 & 4.444 & 0.1096 & -4.715 & -0.1685 \end{bmatrix}$$

$$D_{K\_H\_\inf\_NMP} = \begin{bmatrix} -19.29 & 14.16 \\ 15.59 & -11.44 \end{bmatrix} \tag{25}$$

4. DESIGN OF PSO BASED H-infinity LOOP SHAPING CONTROLLER

Particle Swarm Optimization (PSO) is a familiar algorithm which applies to any type of optimization problem (Randeep Kaur et al., 2014). PSO makes use of fixed cluster of particles called the swarm. All the particles are randomly initialized and entered into iteration process and endorsed to move around to explore the whole search space dimension. Over a number of iterations, each particle exhibits different performance in each iteration considering their present and past values. These particles are guided by previous velocity of each particle, distance from the individual particle's personal best position, and distance from the swarm's global best position. (Febina et al., 2020). The current iteration value is obtained from the highest fitness value of the particle. The PSO parameters inertia weight (Q), value of velocity (v) and position (p) of each particle in the current iteration (i) are updated by using (26), (27) and (28), respectively.

$$Q = Q_{max} - \left(\frac{Q_{max} - Q_{min}}{i_{max}}\right) i \tag{26}$$

$$v_{i+1} = Qv_i + \alpha_1[\gamma_{1i}(P_b - p_i) + \alpha_2[\gamma_{2i}(U_b - p_i) \tag{27}$$

$$p_{i+1} = p_i + v_{i+1} \tag{28}$$

Where, α_1 and α_2 are acceleration coefficients, γ_{1i} and γ_{2i} are any random numbers in (0-1) range. In this paper, a technique is proposed using PSO to find the values of weighting functions to synthesize controller. The proposed PSO technique provides a solution within a short time to calculate the weighting functions for an H-infinity loop shaping controller.

4.1. Weight Selection using PSO Algorithm

The most essential step in the H-infinity loop shaping controller is the selection of weighting functions. Few researchers found that the selection principally influenced by the plant model. Generally, the performance and robustness of the weighting functions are selected by trial-and-error methods. Hence, it is the most challenging to satisfy all the performance criteria of the robust controller simultaneously. Though there are no specific approaches available for the choice of calculating weighting functions, certain generalization can be adopted from the loop shaping procedure (McFarlane et al., 1992). The value of the selected function (stability margin) can be used as a signifying parameter to indicate the quality of the selected weight to satisfy the robust stability criteria. Though in certain instances, the closed-loop system response of a nominal plant in the time domain is not fulfilled, but still ε_{opt} is satisfied in this work, the efficiency parameters are specified and the optimal weight of pre-compensator W_1 is calculated using the PSO. The fitness feature for weight selection is given as fitness = ε_{opt} to meet performance specifications.

4.2. H-infinity Controller Synthesis using PSO Algorithm

The controller(K) designed through the proposed technique is fixed with a structure and to achieve the optimal stability range, the PSO is applied to determine the optimum parameter p. In the proposed method, the stability margin (ε) is a unique index that determines the stable output of K∞ by

$$K_\infty = W_1^{-1} K W_2^{-1} \tag{29}$$

Assume that W_1 and W_2 are invertible. The objective function is:$\varepsilon = \|T_{zw}\|_\infty$

$$= \left\| \begin{bmatrix} I \\ W_1^{-1}K(p) \end{bmatrix} \left(1 - G_s W_1^{-1} K(p)\right)^{-1} [I G_s] \right\|_\infty^{-1} \tag{30}$$

The system parameters p is represented as a particle by means of the PSO technique and the fitness can be written as:

$$\varepsilon = \|T_{zw}\|_\infty = \left\| \begin{bmatrix} I \\ W_1^{-1}K(p) \end{bmatrix} \left(1 - G_s W_1^{-1} K(p)\right)^{-1} [I G_s] \right\|_\infty^{-1} \tag{31}$$

The controller (K) is designed such that the infinity norm is to be minimized from the load disturbances and to maximize the stability margin using the PSO method. Every particle value of fitness or objective function is measured in each iteration of the PSO algorithm. The PSO parameters are presented as in Table 3.

Table 3. PSO parameters.

Operating points	Parameters	Value (Min and Max)
Minimum phase	C1	0.01 and 1
	C2	0.1 and 20
	C3	0.0001 and 0.01
	C4	0.001 and 0.3
	Number of particles	20
	Number of Iterations	20
Non-Minimum phase	C1	10 and 25
	C2	5 and 17
	C3	0.001 and 0.03
	C4	6 and 20
	Number of particles	20
	Number of Iterations	20

5. DESIGN OF GWO BASED H-infinity LOOP SHAPING CONTROLLER

GreyWolf Optimization (GWO) algorithm (Mirjalili et al., 2014; Mirjalili et al., 2015; Mirjalili et al., 2016) is a latest and popular swarm intelligence based algorithm which is developed in the year 2014 by Mirjalili et al. The GWO algorithm mimics the control structure and the method of shooting of the grey wolves in the mountains. There are four classes of simulations that are applicable in grey wolf structure as presented in Figure 3 (P.B. de Moura Oliveira et al., 2016; Madadi et al., 2014).

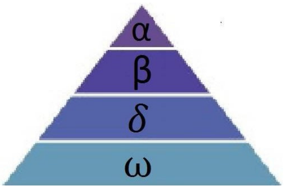


Fig. 3. The Grey Wolf Hierarchy.

The *Alpha (α)* wolf are the leaders of the entire group which are highly responsible for decision making about various mechanisms like hunting, sleeping place, time to wake, etc. The subordinate of *Alpha (α)* wolves is *Beta (β)* which is at the second rank of hierarchy assist the *Alpha (α)* in decision making for hunting mechanism and other activities (Verma et al., 2017; Lal et al., 2016; Tsai et al., 2016) Generally, the solution searches in this algorithm initiates with a population of wolves(solutions) are generated in a random manner. These wolves estimate the position of the prey (optimum) through iterative methods during the hunting (optimization) process. Alpha (α) is the most suitable solution preceded by Beta (β) and Delta (δ) as the second and third best solutions. The remaining options are of the least value and are called Omega (*ω*) (Shahrazad et al., 2015). The following equations are suggested to mathematically modulate the encircling behavior:

$$\vec{D} = |\vec{C}X_P(t) - \vec{X}(t)| \quad (32)$$

$$\vec{X}(t+1) = |\overrightarrow{X_P}(t) - \vec{A}\vec{D}| \quad (33)$$

If t is the current iteration, $\vec{A}$ is a coefficient vector, and $\vec{C}$ is a coefficient variable, $\overrightarrow{X_P}(t)$ is the victim's position variable. X suggests the place vector of the gray wolf. The current iteration vectors, $\vec{A}$ and $\vec{C}$, are calculated as follows:

$$\vec{A} = 2\vec{a}\vec{r}_1 - \vec{a} \quad (34)$$

$$\vec{C} = 2\vec{r}_2 \quad (35)$$

The following methods were provided for upgrading the locations of the best search agents.

$$\overrightarrow{D_\alpha} = |\overrightarrow{C_1}X_\alpha - \vec{X}|, \overrightarrow{D_\beta} = |\overrightarrow{C_2}X_\beta - \vec{X}|, \overrightarrow{D_\delta} = |\overrightarrow{C_3}X_\delta - \vec{X}| \quad (36)$$

$$\vec{X} = \overrightarrow{X_\alpha} - \overrightarrow{A_1}\vec{D}_\alpha, \vec{X} = \overrightarrow{X_\beta} - \overrightarrow{A_1}\vec{D}_\beta, \quad \vec{X} = \overrightarrow{X_\alpha} - \overrightarrow{A_1}\vec{D}_\delta \quad (37)$$

$$\vec{X}(t+1) = \frac{\overrightarrow{X_1} + \overrightarrow{X_2} + \overrightarrow{X_3}}{3} \quad (38)$$

The GWO algorithm parameters chosen are shown in Table 4.

Table 4. Grey Wolf Optimizer Parameters.

Operating points	Minimum Phase	Non-Minimum Phase
Maximum iteration	20	20
Population size	20	25
Number of Decision variables	04	04

6. SIMULATION AND RESULTS

In order to evaluate whether the controller was able to retain set point tracking functionality, reference signal was applied to the system as a step function. Programming was done using MATLAB\Simulink (version 9.6) (Appendix A and B) in order to investigate the various performances of quadruple tank system with respect output level 1 and 2, respectively. Due to optimization algorithms, it has to be run several times to get a statistical validation and evaluation of its performance (each of them involving 20 iterations). The parameters were adjusted using GA, PSO, and GWO based searching and the best values were updated in every iteration. From the computed values, optimal weight pre-compensator W_1 was determined for H-infinity controller whereas post-compensator W_2 was chosen as constant value (W_2=1). The performance of the H-infinity loop shaping controller in application to quadruple tank system was examined both in minimum and non-minimum phase using calculated optimal weights. It is observed that all the control schemes are able to track the changes and retain a desired setpoint despite the changes in comparison. The optimization algorithms GA, PSO, GWO were used to search the minimum value of the cost function to reduce h-infinity norm. The system was simulated with tuned weights of the H-infinity controller and the results are shown for both minimum and non-minimum phase in the Figures (4-11). `

The servo response and regulatory response of the system can be easily enhanced by varying the gamma value γ as shown in Figures 4 and 5. The cost function (chosen as minimum value), gamma is calculated as 1.44, 1.411, 1.4,1.338 for H-infinity, GA, PSO, GWO based H-Infinity controllers, respectively for level 1 and level 2 of the tank in minimum phase. Slothful response of the tank is observed for higher value of γ and good servo response and disturbance rejection is observed for lower value of γ.

From the Figures 6 and 7, it is proved that the overall performance of the system can be easily altered for non-minimum phase with the help of gamma value. The gamma values are calculated as 30, 28.11, 26.19, 14.25, respectively for for H-infinity, GA, PSO, GWO based H-Infinity controllers, respectively for level 1 and level 2 of the tank in non-minimum phase. As seen in Figures 4, 5, 6, and 11, H-infinity controllers that are tuned using the GWO algorithm guarantee both regulatory response and servo response than GA and PSO algorithm in both minimum and non-minimum phases. As can be seen, GWO algorithm converges faster at the end than the PSO and GA algorithms. In other words, the GWO algorithm provides better strategies for identifying weights relative to the GA and PSO algorithms in terms of set point monitoring, load disturbance rejection, and plant instability robustness.

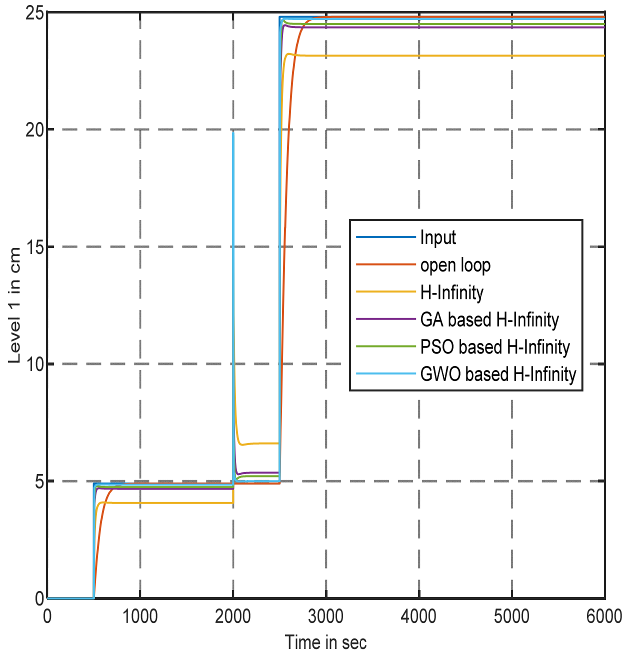


Fig. 4. Responses of Minimum Phase QTS for Servo and Regulatory Operation Applied to Loop 1(Level 1).

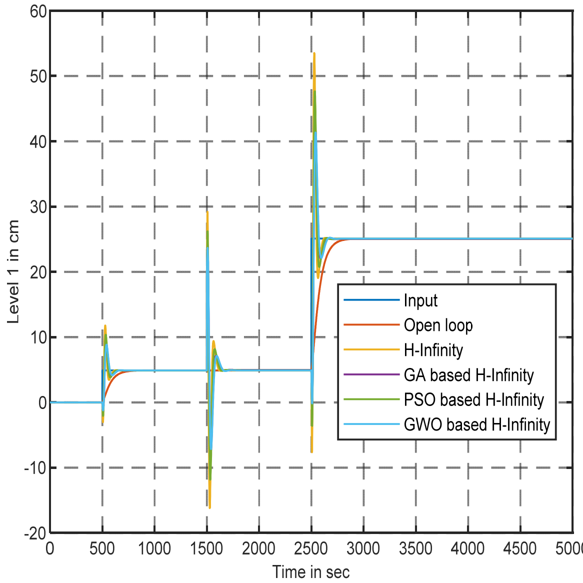


Fig. 5. Responses of Minimum Phase QTS for Servo and Regulatory Operation Applied to Loop 2 (Level 2).

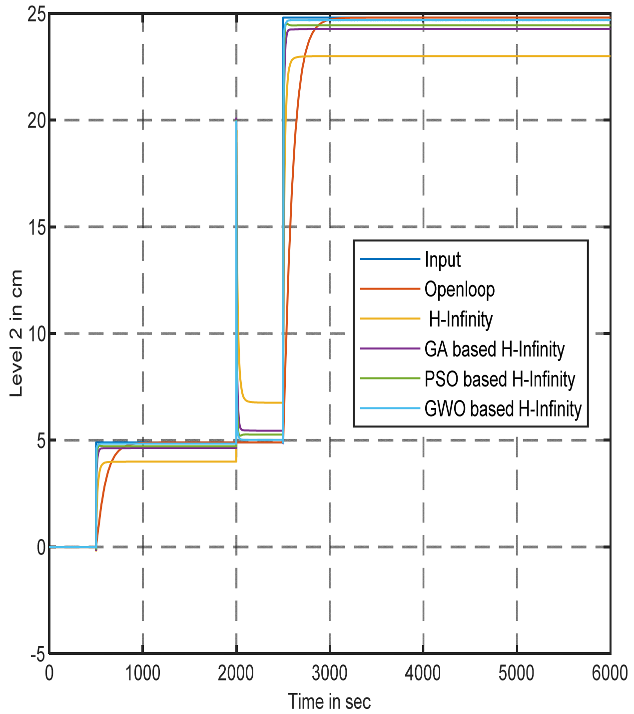


Fig. 6. Responses of Non-minimum Phase QTS for Servo and Regulatory Operation Applied to Loop 1(Level 1).

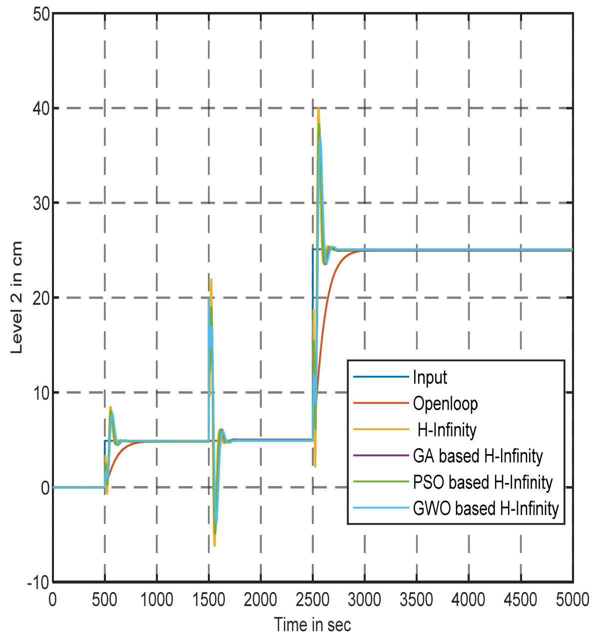


Fig. 7. Responses of Non-minimum Phase QTS for Servo and Regulatory Operation Applied to Loop (Level 2).

The optimal weights thus calculated from the optimization algorithms such as GA, PSO and GWO are used to shape the open loop response of the quadruple tank both in minimum and non-minimum phase for level 1 and 2, respectively.

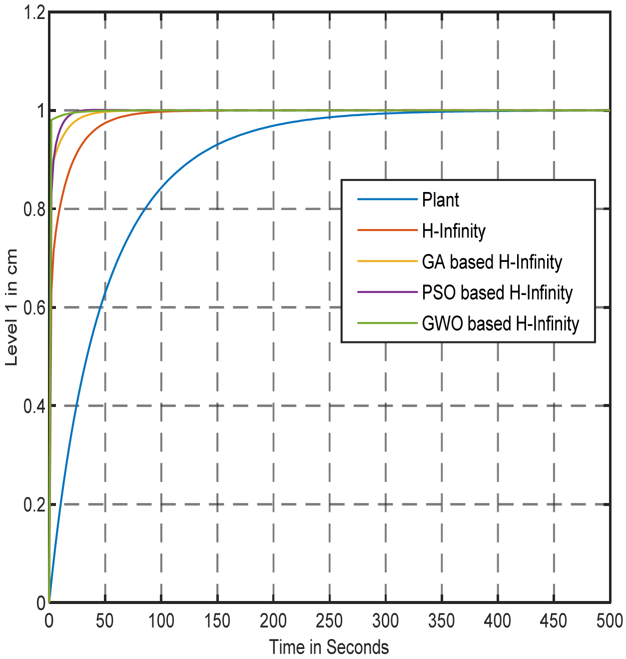


Fig. 8. Open Loop-shaped Response of Loop 1 in the Minimum Phase.

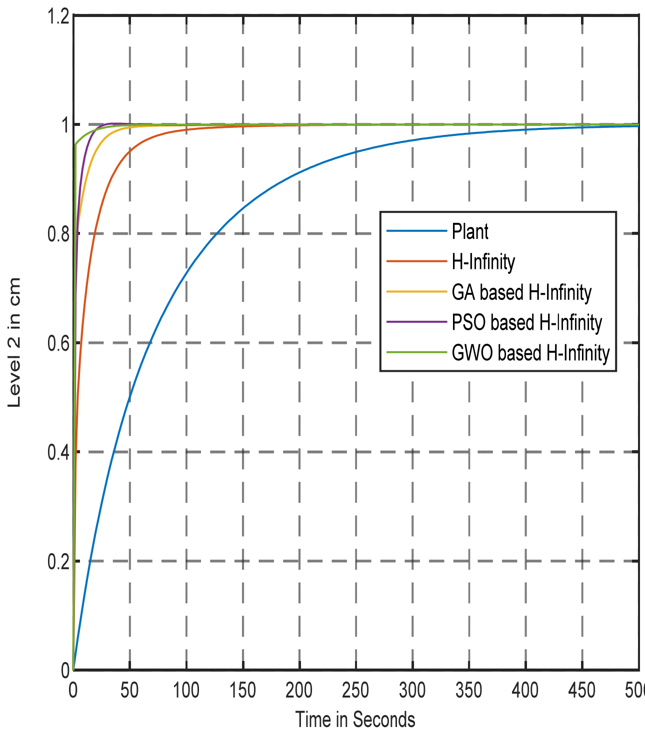


Fig. 9. Open Loop-shaped Response of Loop 2 in the Minimum Phase2.

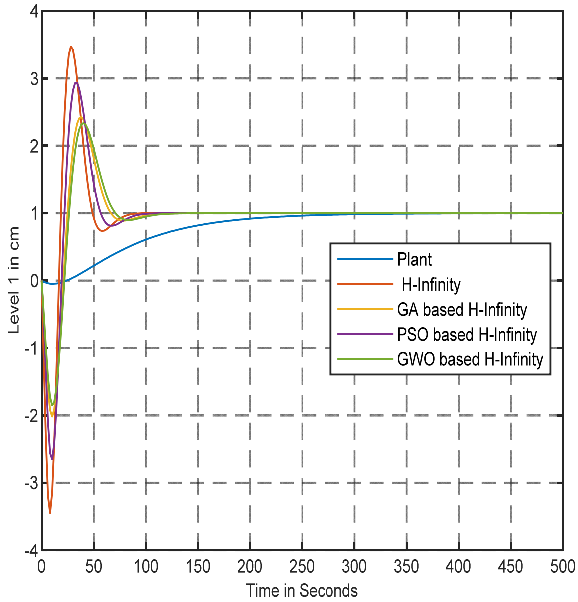


Fig. 10. Open Loop-shaped Response of Loop 1 in Non-Minimum Phase.

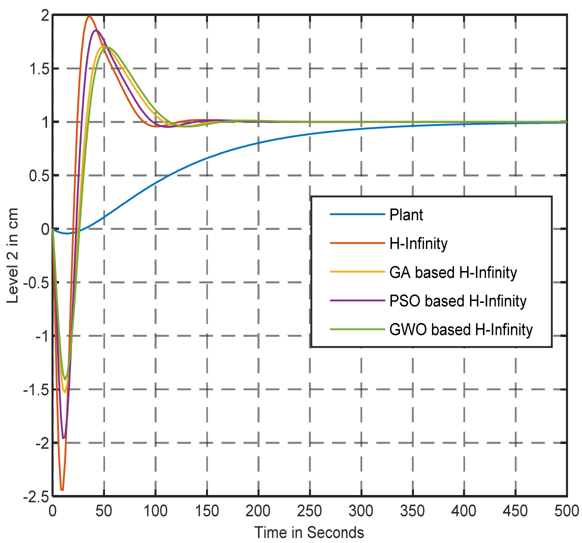


Fig. 11. Open Loop-shaped Response of Loop 2 in Non-minimum Phase.

The Figures 8-11 portrayed that optimal weight selection methods has drastically improved the open loop shape of the quadruple tank system with transmission zero. The performance measures of the designed H-infinity controller, PSO tuned H-infinity controller and GWO tuned H-infinity controller such as overshoot, risetime, settling time and integral error are compared and tabulated in Table 5.

Table 5. Comparison of controller performance measures.

Operating points		Parameters	Controller			
			HILSC	GA tuned HILSC	PSO tuned HILSC	GWO tuned HILSC
Minimum phase	Level 1	Gamma	1.422	1.41	1.4	1.338
		Integral square error(ISE)	550.5	480.5	244.7	51.84
		RiseTime (in sec)	4.093	3.888	1.935	1.292
		Overshoot(%)	190.38	160.2	96.91	91.51
	Level 2	Gamma	1.422	1.411	1.4	1.338
		Integral square error(ISE)	850.9	644.5	414.4	86.9
		Rise Time (in a sec)	4.67	3.77	2.680	2.294
		Overshoot (%)	190.38	139.2	96.91	91.51
Non-minimum phase	Level 1	Gamma	30	28,11	26.19	14.65
		Integral square error (ISE)	5668	3445	746.7	487
		Rise Time (in a sec)	3.27	3.1	2.945	9.406
		Overshoot (%)	143.019	100.9	229.892	99.98
	Level 12	Gamma	30	28.11	26.19	14.65
		Integral square error (ISE)	459	443	428.6	467.9
		Rise time Time (in sec)	4.403	18.99	20.405	30.434
		Overshoot (%)	227.204	222.6	221.992	105.74

7. CONCLUSIONS

In this paper, the H-infinity Loop Shaping Controller (HILSC) for the quadruple tank system is designed to ensure a multi-loop control in both minimum and non-minimum phase using the classical method, GA, PSO, and GWO algorithms and simulations were examined. The significance of this work is that for the first time, GWO is used as a variety method for a quadruple tank system to adjust H-infinity controllers. In comparison, to analyze the output of GWO, it is contrasted with a common search algorithm such as GA and PSO. The GWO algorithm was effectively used for the configuration of H-infinity controllers. The suggested algorithms are used to determine the optimum weights for optimal controller output in the quadruple tank system by determining the minimum value of gamma easily. The findings of the simulation reveal that the GWO algorithm is faster and more effective than the GA and PSO algorithms in the global and local optimization search. In this study, the GWO algorithm is the best that demonstrates the satisfactory performance of the GA and PSO algorithms. Nevertheless, the tuned weights of the H-infinity controllers performed servo and regulatory control of the system productively with all the proposed algorithms. It is anticipated that this method will contribute more to the quadruple tank system that works about most of the multivariable systems especially with the non-minimum phase. The time-domain specifications analysis of the designed controller has produced better results. Hence, the proposed controller can be applied to any multivariable system with control challenges.

REFERENCES

Corriou, Jean-Pierre, (2018)"Multivariable Control by Transfer Function Matrix." *In-Process Control*, pp. 305-338. Springer, Cham, 2018.

De Moura Oliveira. P.B., Freire.H, and SolteiroPires.E.J (2016), "Grey wolf optimization for PID controller design with prescribed robustness margins", *Soft Computing*, vol.20, pp.4243-4255

C. Febina*, D. Angeline Vijula(2020), "Model based Controller Design using Real Time Neural Network Model and PSO for Conical Tank System", *CEAI*, Vol.22(3), pp. 13-24

Genc.U, (2000), "A state-space algorithm for designing H-infinity loop shaping PID controllers, "technical report., *Cambridge University*, Cambridge, UK

Ho.S.J, Ho.S.Y, Hung.M.H, Shu.L.S, et al.,(2005), "Designing structure-specified mixed H2/ H∞ optimal controllers using an intelligent genetic algorithm IGA, *IEEE Trans. On Control Systems*, 13(6), pp.1119-24.

Johansson, K H (2000), "The quadruple-tank process: A multivariable laboratory process with an adjustable zero," *IEEE Transactions on control systems technology* Vol.8, No. 3, pp. 456-465.

Kirubakaran, V., Radhakrishnan. K., and. Sivakumaran N (2014). "Distributed multiparametric model predictive control design for a quadruple tank process." *Measurement* Vol. 47, pp. 841-854.

La.D.K, A. K. Barisal, and M. Tripathy(2016), "Grey wolf optimizer algorithm based fuzzy PID controller for AGC of multi-area power system with TCPS", *Procedia Computer Science,* vol. 92, pp. 99-105,.

Madadi.A and Motlagh.M.M(2014), "Optimal control of DC motor using grey wolf optimizer algorithm", *TJEAS Journal*-2014-4-04/373-379, vol. 4, no. 4, pp.373-79.

McFarlane, D., Glover, K. (1992). A loop shaping design procedure using H∞ synthesis. *IEEE Transactions on Automatic Control*, 37(6), pp. 759-769.

Mirjalili, S. Mirjalili,M (2014), and A. Lewis, "Grey wolf optimizer", *Advances in Engineering Software*, vol. 69, pp. 46-64.

Mirjalili.S et al. (2016), "Multi-objective grey wolf optimizer: a novel algorithm for multi-criterion optimization", *Expert Systems with Applications* vol. 47, pp.106-119, 2016.

Mirjalili S (2015), "How effective is the Grey Wolf optimizer in training multilayer perceptrons, *Applied Intelligence*", vol. 43, no. 1, pp. 150-161, 2015.

Osinuga, M., Patra, S., &Lanzon, A. (2012). The state-space solution to weight optimization the problem in H1 loop-shaping control. *Automatica*, 48(3), 505–513.

Randeep Kaur, JyotiOhri (2014) "PSO Based Weight Selection and Fixed Structure Robust Loop Shaping Control for Pneumatic Servo System with 2DOF Controller", World Academy of Science, *Engineering and Technology International Journal of Electrical and Computer Engineering* Vol:8, No:8.

Salim.M and Khosrowjerdi.M.J (2017) "Data-driven H∞ Controller/Detector Design for a Quadruple Tank Process" *Control Engineering and Applied Informatics*, Vol.19, No. 1, pp. 3-14.

Shahrzad, S. Mirjalili,. Z and Mirjalili.M, (2015) "Evolutionary population dynamics and grey wolf optimizer", *Neural Computing and Applications* vol. 26, no. 5, pp. 1257-1263.

Shenhua Yang, Minjie Zheng (2018), "H∞ Fault-Tolerant Control for Dynamic Positioning Ships based on Sampled-data" *Control Engineering and Applied Informatics*,Vol. 20, No.4 pp. 32-39.

Shneiderman, D., and Palmor.Z. J (2010), "Properties and control of the quadruple-tank process with multivariable dead-times." *Journal of Process Control* Vol. 20, No. 1, pp. 18-28.

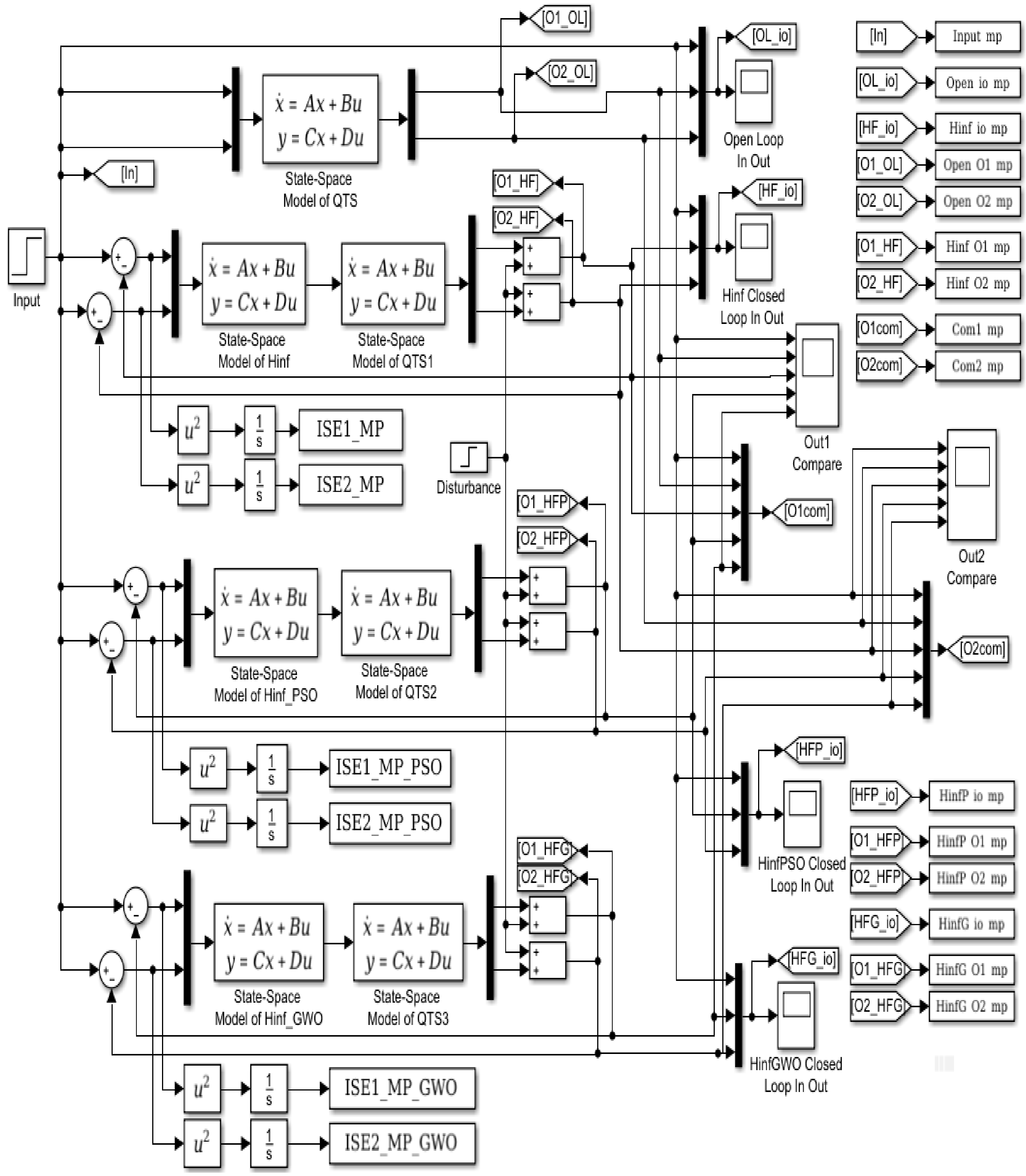
Skogestadt S. and Postlethwaite I, Multivariable Feedback control, *Wiley*, 1996

Tsai.P.W,Nguyen.T.T,Dao.T.K(2017), "Robot Path Planning Optimization Based on Multiobjective Grey Wolf Optimizer", In Pan JS., Lin JW., Wang CH., Jiang X. (eds) Genetic and Evolutionary Computing. ICGEC 2016. *Advances in Intelligent Systems and Computing, Springer*, Cham, vol 536, pp.166-173.

Verma.S.K, S. Yadav.S, and Nagar.S.K, (2017)"Optimization of Fractional Order PID Controller Using Grey Wolf Optimizer", *Journal of Control Automation and Electrical systems*, vol. 28, pp.

APPENDIX A

Simulink block diagram of quadruple tank system in minimum phase:



APPENDIX B

Simulink block diagram of quadruple tank system in non-minimum phase: