Grey Wolf Optimization Algorithm Based Weight Selection for Tuning H-Infinity Loop Shaping Controller in Application to A Benchmark Multivariable System with Transmission Zero

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Abstract: This paper presents a method based on Grey-Wolf Optimization(GWO) algorithm based algorithm for the H-infinity loop shaping controller(HILSC) design for a multivariable system in non-minimum phase and minimum phase The role of GWO is to determine the weighing function parameters of the HILSC by minimizing the norm (H-infinity) of the CLTF of the multivariable process which is a quadruple tank in this case. The optimal stability margin in an HILSC is taken as objective function of GWO to compute the utmost appropriate weighting parameters. The mathematical equivalent for non-minimum phase and minimum phase of quadruple tank is obtained in state-space form by moving the operating position of the valve and the benchmark HILS procedure is employed to determine the weighting functions of the HILSC. The difficulties usually arise in finding compensators that implement most design decisions. It is revealed that the proposed method subsequently reduces the technical hitches in the designing method of H-infinity controller for a multivariable system(MVS). GWO based optimization provides comparatively better results than PSO based optimization for a quadruple tank, which is evident in the simulation resultants obtained against plant perturbations and disturbances. The integral square error is reduced appreciably with the proposed technique. The necessary computations and the analysis of the behavior of the system are done in MATLAB simulation environment.

Keywords: Grey Wolf Optimization (GWO), H-infinity loop shaping controller (HILSC), Multivariable control system, HILS (H-infinity loop shaping), CLTF (Closed-Loop Transfer Function), Non-minimum phase systems, Multivariable System (MVS), Particle Swarm Optimization (PSO).

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1. INTRODUCTION

Most of the complicated engineering structures are prepared with many actuators which can influence their behavior (dynamic and static as well). In cases where few shapes of automatic (computerized) control are required over the system, numerous sensors are employed to reveal the information of process variable(s) measurement, which are used for feedback control of the system. A system is said to be an MVS or a MIMO (Multi-Input-Multi-Output) system when it possesses multiple actuating input control / sensor output. The critical goal for an MVS is manipulating the input variables and input from channels of the system, simultaneously to obtain the desired output. Many control problems in process industries are nonlinear and feature more than one controlled variables which can be not unusual properties for mathematically derived models of extensive uncertainties, robust interactions, and non-minimum phase behavior so its miles important for control engineer, chemical engineer to recognize the non-idealities of industrial systems through sporting out experiments with a good laboratory equipment (Shneiderman.D et al .,2010)The quadruple tank's general mathematical model can retain all the properties of the system, but location of the zeros can be determined by the values of $\gamma1$ and $\gamma2$ are set (Corriou Jean-Pierre,2018).The physical system parameters that are determined through mathematical modelling cannot be more accurate. The major issues with mathematical modelling are inaccurate model parameters which may occur due to various factors. The parameter values are subjected to change with the change in time and different characteristics. The uncertainty will exist between the actual system and mathematically modelled system because of their difference (Ho.S.J et al., 2005, McFarlane, D et al., 1992).

To deal with an uncertain plant an appropriate uncertainty model is to be decided on and the complete system dynamics is also taken into consideration. Robust control provides essential techniques based totally upon an incomplete description of the controlled process applicable within the regions of non-linear and time-various approaches, which includes MIMO (multi-input and multi output) dynamic systems (Indirapriyadharshini.J, et al 2016)

Glover and McFarlane proposed a HILS design method (McFarlane. D et al., 1992) consists of open-loop shaping through pair of compensators to achieve overall performance/stability trade-offs. Designers are to follow the design procedure to develop the for their system based on the open loop characteristics for determining the compensators for loop shaping, and a suitable weighting function selection will result in robust controller. Although this approach is properly-organized to design a structure specified robust controller; however, the choice of uncertainty weight of their methods is complex. Hereto, in the MIMO system, the issue of uncertainty in weight selection will become an essential venture to be addressed.

To conquer this drawback of H-infinity optimal control, McFarlane and Glover (McFarlane. D et al., 1992) proposed an approach referred to as H-infinity loop shaping control. This approach is primarily built with the idea of open loop shaping of a system by simply using two compensator weights that want to be decided on. Fortunately, the compensator weight selection way on this technique is very apparent via the classical control theorem. Therefore, there is still a lack of concrete ideas on how to define the weighting characteristics (Shenhua Yang , Minjie Zheng ,2018). One simple motive is that the LSDP itself can deal with a huge variety of control complications and issues based on the systematic selection of weighing functions. It motivates to find such a specific method by placing a restriction on a class of goal control systems.

In this proposed work, GWO is implemented to select optimal weighting functions of structure-specified *H*-infinity loop shaping controller of a multivariable system for minimum and non- minimum phase modes..The quadruple tank process model is initially formed by the pre-compensator (W1) and the post-compensator (W2) in order to achieve the desired open-loop form and to determine the structure of the actual robust controller. Eventually, GWO is used to check for the parameters of the designed controller in a way that the stability margins is reduced. Outcomes of the proposed method have been finally compared with the conventional *H*-infinity loop shaping approach and PSO algorithm. The comparison of the integral square error (ISE) based on the *H*-infinity loop shaping approach; PSO and GWO based algorithms are also included.

2. OPERATION AND MODELING OF QUADRUPLE TANK SYSTEM

The quadruple tank brought by Johansson (Johansson 2000) gained first-rate publicity as it exhibits fascinating characteristics in both teaching and research. The quadruple tank displays complex dynamics elegantly and in reality. Those dynamic characteristics include interactions and a zero-place transmission that can be modified in operation. With good enough tuning, this system provides non-minimum phase characteristics that arise because of the MIMO nature of the system. And so, the quadruple tank was used to elucidate the effects of different control techniques and as a training device in teaching superior MIMO system characteristics and their control methods. (Salim.M and Khosrowjerdi.M.J ,2017).

The four tank are connected with tow pumps and formed the structure of quadruple tank process which is presented in Figure 1. The parameters v1 and v2 (enter voltages to the pumps) are considered as process inputs and parameters y1 and y2 (voltages from level measuring devices) are considered as process outputs. The quadruple tank system is designed such that the desired level of lower tanks is achieved by inlet flow rate variation. Diagonally in opposite position,the flow fr

Therefore, each pump flow goes to 2 tanks, one decrease and any other upper one, diagonally opposite, and the break up ratio is controlled by the three-way valve position. With any adjustment in the location of the 2 valves, the mechanism can be either within the minimum phase or within the non-minimal phase, as it should be. (Biswas, PinakPani et al.,2009, Kirubakaran, et al., 2014)

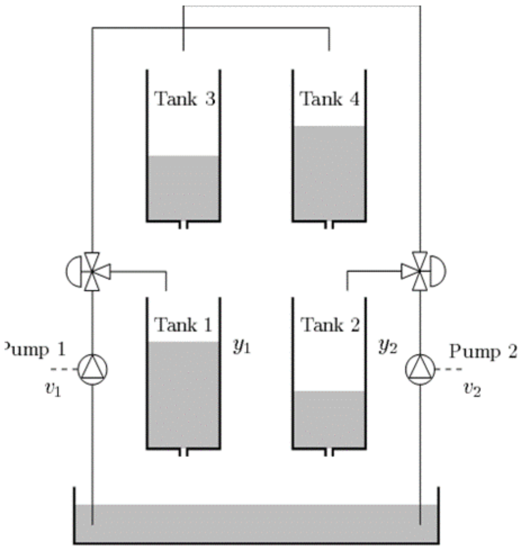

Fig. 1. Quadruple Tank Process

Let the parameter γ be calculated by the setting of the valves. When γ_1 is the flow ratio to the first tank, the flow to the fourth tank will be (1-γ_1). Likewise, if γ2 is the flow ratio to the second tank, then the flow to the third tank will be (1-γ_2). The voltage added to the '*i*' pump is *V*i, and *KiVi* is the equivalent flow rate.

Based on how the valves are placed before an experiment, the parameters γ_1, γ_2 ε (0, 1) are calculated. The flow to the' 1' tank is $\gamma_1 K_c V_1$ and the flow to the' 4' tank is $(1-\gamma_1)K_c V_1$ and similarly to the' 2' and' 3' tank. Gravity force is referred to as' g.' The determined signal rates $y_1 = k_c h_1$ and $y_2 = k_c h_2$.$k_{c\ are}$ the co-efficient of pump discharge. The equations (1)-(4) represents the quadruple tank system in non-linear form as

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_c}{A_1} v_1 \qquad (1)$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_c}{A_2} v_2 \qquad (2)$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_c}{A_2} v_2 \qquad (3)$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_c}{A_2} v_2 \qquad (4)$$

where

A_i is the cross-sectional area of Tank 'i'

a_i is a cross-section of outlet hole of Tank 'i'
h_i is the water level in Tank 'i'.

The tank levels (hi) of all four tanks in the quadruple tank cycle are used as the state variables (xi), the voltages added to pumps (v1 and v2) are the input variables (ui), and the output variables (yi) are the amounts of tank ' 1 ' and tank ' 2).

For linearization of the non-linear method, and $xi = hi\text{-}hi^0$; $ui = vi\text{-}vi^0$ where hi^0 and vi^0 are the *hi* and *vi* defined values. The linearized state-space model with function matrices is obtained using the expansion of the Taylor series set out in (5) and the configuration is given in (6) and (7).

$$\dot{x_i} = \dot{h_i} \approx f(h_i^0, v_i^0) + \frac{\partial f(h,v)}{\partial h^T} /_{h_i = h_i^0} \; (h_i - h_i^0) + \frac{\partial f(h,v)}{\partial v^T} /_{v_i = v_i^0} \; (v_i - v_i^0) \tag{5}$$

$$f(h_i^0, v_i^0) \xrightarrow{yields} 0; \frac{\partial f(h,v)}{\partial h^T} \xrightarrow{yields} \boldsymbol{A}; \tag{6}$$

$$\frac{\partial f(h,v)}{\partial h^T} /_{h_i = h_i^0} \xrightarrow{yields} \boldsymbol{X}; \tag{7}$$

$$\frac{\partial f(h,v)}{\partial v^T} /_{v_i = v_i^0} \xrightarrow{yields} \boldsymbol{B}; \tag{8}$$

$$(v_i - v_i^0) = \xrightarrow{yields} \boldsymbol{U}; \tag{9}$$

The linearized model is given by

$$\begin{aligned} X &= A\dot{X} + BU \\ Y &= CX + DU \end{aligned} \tag{10}$$

where

$$A = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & 0 \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} \; and \; D = 0; State \; vector, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

The time constant is calculated from the equation

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}, i = 1, \dots, 4 \tag{11}$$

The steady state operating points and process parameter values are assumed as given in the literature (Johansson 2000), and the values are tabulated respectively in Table 1 and Table 2.

Table 1. Process parameters

Area of Tank 1 and 3 (A_1, A_3)	28 cm^2
Area of Tank 2 and 4 (A_2, A_4)	32 cm^2
a_1, a_3	0.071 cm^2
a_2, a_4	0.057 cm^2
K_c	0.5 V/cm

Table 2. Steady state operating points

Steady state parameters	Minimum Phase	Non minimum Phase
h_1^0, h_2^0 [cm]	(12.4 , 12 .7)	(12 .6 , 13)
h_3^0, h_4^0 [cm]	(1 .8 , 1 .4)	(4 .8 , 4 .9)
v_1^0, v_2^0 [V]	(3.00 , 3 .00)	(3 .15 , 3 .15)
k_1, k_2 [cm^3/V s]	(3..33 , 3.35)	(3 .14 , 3 .29)
γ_1, γ_2	(0 .70 , 0.60)	(0 .43 , 0 .34)

The state-space matrices A, B, and C are determined by removing the values above in (11). The transfer function matrices are obtained using the MATLAB function and are given in (12) and (13) respectively for minimum phase and non-minimum phase operating points.

$$G_{\min(s)} = \begin{bmatrix} \frac{2.6}{1+62s} & \frac{1.5}{(1+23s)(1+62s)} \\ \frac{1.4}{(1+30s)(1+90s)} & \frac{2.5}{1+90s} \end{bmatrix} \tag{12}$$

$$G_{\text{non\_min}(s)} = \begin{bmatrix} \frac{1.5}{1+63s} & \frac{2.5}{(1+39s)(1+63s)} \\ \frac{2.5}{(1+56s)(1+91s)} & \frac{2.5}{1+91s} \end{bmatrix} \tag{13}$$

The generated transfer matrix G (12) and (13) usually has two zeros in which one of the zeros is always positioned in the left half of the s-plane, while another zero can be put either in the left half or in the right half of the s-plane, depending on the location of the three-way valve. As a consequence, the mechanism is said to be in the minimum phase if the valves γ_1 and γ_2 are in the $0 < \gamma_1 + \gamma_2 < 1$ and non-minimum phase, if the valves γ_1 and γ_2 are in the position $1 < \gamma_1 + \gamma_2 < 2$.

3. DESIGN OF H-Infinity LOOP SHAPING CONTROLLER

The figure 2 (where *s* is the Laplace variable) shows the general closed loop structure of a system.

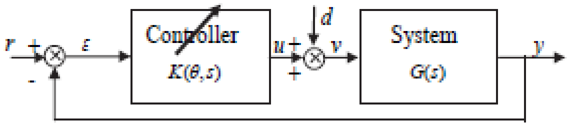

Fig. 2. Classic controller framework

The plant is modelled with the transfer function G(s) and it has to be controlled to get desirable performances. The input and output of the controller is v and y respectively. The performance of this controller *K* (*s*, θ) is depending upon the parameter θ to be selected as optimal. Generally, the objective of designing closed-loop system is to ensure that the system output *y* follows set point *r* tracking and is very stable against the load disturbances *d*. The role of the controller is the output parameters of this closed-loop device (actually complicated and found in the y, u and/or ε to some responses to specific inputs added to the r or d inputs).

McFarlane and Glover (McFarlane. D et al., 1992) suggested an appropriate method for the configuration of the H-infinity loop shaping mechanism, which was a solution to a wide range of control problems. The uncertainties are represented in this approach as co-prime uncertainty and this concept does not reflect actual physical uncertainty.W1 (precompensator) and W2 (postcompensator) are the two weighting functions that set out to shape the real plant G in order to achieve the desired open-loop structure. The shaped plant obtained by this method is expressed as

$$G_s = W_1 G W_2 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{14}$$

$$G_s = (N_s + \Delta_{NS})((M_s + \Delta_{MS})^{-1} \tag{15}$$

where *A, B, C, D* represent plant *Gs* in the state-space form,

$$\|\Delta_{NS}, \quad \Delta_{MS}\|_\infty \leq \varepsilon \tag{16}$$

where Ns and Ms are nominators and denominators of generalized coprime variables. Δ_{NS} and Δ_{MS} are ambiguity mapping mechanisms with nominator and denominator variables. ε is an uncertainty boundary, called a stability margin (McFarlane. D et al., 1992).

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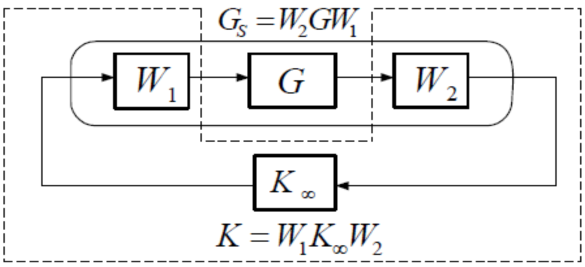

Fig. 3. Block Diagram of H-infinity controller

The following steps are suggested for the design of a typical H-infinity loop shaping controller based on (McFarlane. D et al., 1992):

1. The selection of *W*1 and *W*2 such that the shaped plant *Gs* has no hidden approaches. The precompensator W1 is usually chosen to monitor output and interference rejection, and W2 is chosen to reduce sensor noise. Essentially, W1 is chosen as a weighting function with an essential action to make a zero steady-state error The post compensator *W*2 can be selected as an identity matrix to remove the noise influence of the system when the right sensor is used. For minimum phase:

$$W_1 = K_w \frac{s+a}{s+b} = \frac{2s+0.1}{0.04s+0.002} \tag{17}$$

For the non-minimum phase:

$$W_1 = K_w \frac{s+a}{s+b} = \frac{12s+12}{12s+0.015} \tag{18}$$

where K_w and a and b are positive numbers in which b provides integral action with the value chosen as (<<1) .

2. The infinity -norm is to be minimized for designing overall stabilizing controllers' *K* and the optimal cost γ_{opt} is calculated as,

For minimum phase:

$$\gamma_{opt} = \varepsilon^{-1}{}_{opt} = \inf \left\| \begin{bmatrix} I \\ K \end{bmatrix} (1 + G_s K)^{-1} M_s^{-1} \right\|_\infty = 1.528 \tag{19}$$

For the non-minimum phase:

$$\gamma_{opt} = \varepsilon^{-1}{}_{opt} = \inf \left\| \begin{bmatrix} I \\ K \end{bmatrix} (1 + G_s K)^{-1} M_s^{-1} \right\|_\infty$$

$$= 30.775 \tag{20}$$

3. Choose $\varepsilon < \varepsilon_{opt}$, and evaluate controller K_∞. The state model of the H-infinity controller is given as follows:

For minimum phase:

$$A = \begin{bmatrix} -0.03743 & -0.006521 & 0.02562 \\ 0.003425 & -0.1896 & -0.01235 \\ -0.02082 & -0.02413 & -0.04583 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.04783 & -0.1299 \\ 0.3632 & -0.2446 \\ 0.04297 & -0.02071 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.0633 & 0.3253 & 0.02398 \\ 0..09166 & -0.2598 & -0.01056 \end{bmatrix}$$

$$D = \begin{bmatrix} -0.7777 & 0.5063 \\ 0.5708 & -0.3716 \end{bmatrix} \tag{21}$$

For Non-minimum phase,

$$A = \begin{bmatrix} -0.154 & 0.3062 & -0.1839 & -0.3508 & -0.04948 \\ -0.07386 & -2.918 & -0.1102 & 3.282 & 0.1133 \\ 0.07244 & -0.256 & -0.07941 & 0.2927 & 0.04634 \\ -0.04896 & -1.81 & 0.07437 & 1.76 & -0.0605 \\ 0.01868 & -0.1383 & -0.03797 & 0.1408 & -0.05669 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.637 & -7.705 \\ 10.47 & -0.6085 \\ 0.9965 & -4.538 \\ 6.185 & -4.538 \\ 0.4919 & -0.3636 \end{bmatrix}$$

$$C = \begin{bmatrix} -0.07843 & 5.425 & -0.2746 & -5.85 & -0.2053 \\ -0.3311 & -4.444 & -0.1094 & 4.715 & 0.1858 \end{bmatrix}$$

$$D = \begin{bmatrix} -19.29 & 14.16 \\ 15.59 & -11.44 \end{bmatrix} \tag{22}$$

4. Final overall controller (K) follows

$$K = W_1 K_\infty W_2 \tag{23}$$

The state model of the final overall controller is designed as follows:

For Minimum phase:

$$A = \begin{bmatrix} -0.03743 & 0 & -0.0633 & -0.3253 & -0.0.2398 \\ -0.05 & -0.05 & -0.09166 & 0.2598 & 0.01056 \\ 0 & 0 & -0.03743 & -0.006521 & 0.01056 \\ 0 & 0 & 0.003425 & -0.1896 & -0.01235 \\ 0 & 0 & -0.02082 & -0.02413 & -0.04583 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.7777 & -0.5063 \\ 0.5708 & 0.3716 \\ -0.04783 & -0.1299 \\ 0.3632 & -0.2446 \\ 0.04297 & -0.02071 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & -0.03165 & -0.1627 & -0.01199 \\ 0 & 0 & -0.04583 & 0.1299 & 0.00528 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.3888 & -0.2531 \\ -0.2854 & 0.1858 \end{bmatrix} \tag{24}$$

For the non-minimum phase:

$$A = \begin{bmatrix} -0.00125 & 0 & 0.07843 & -5.425 & -0.2756 & 5.85 & 0.2053 \\ 0 & -0.00125 & 0.3311 & 4.444 & 0.1094 & -4.715 & -0.1685 \\ 0 & 0 & -0.1534 & 0.3062 & -0.1839 & -0.3508 & 0.04948 \\ 0 & 0 & -0.07386 & -2.918 & -0.1102 & 3.282 & 0.1133 \\ 0 & 0 & 0.07244 & -0.256 & -0.07941 & 0.2927 & 0.04634 \\ 0 & 0 & -0.04896 & -1.81 & -0.07437 & 1.76 & 0.0605 \\ 0 & 0 & 0.01868 & -0.1383 & -0.03797 & 0.1408 & -0.05669 \end{bmatrix}$$

$$B = \begin{bmatrix} 19.29 & -14.16 \\ -15.59 & 11.44 \\ -0.637 & -4.538 \\ 10.47 & -4.538 \\ 0.9965 & -0.3636 \\ 6.185 & -4.538 \\ 0.4919 & -0.3636 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.9988 & 0 & 0.07843 & -5.425 & -0.2756 & 5.85 & 0.2053 \\ 0 & 0.9988 & 0.3311 & 4.444 & 0.1096 & -4.715 & -0.1685 \end{bmatrix}$$

$$D = \begin{bmatrix} -19.29 & 14.16 \\ 15.59 & -11.44 \end{bmatrix} \tag{25}$$

4. DESIGN OF PSO BASED H-infinity LOOP SHAPING CONTROLLER

Particle Swarm Optimization (PSO) is a familiar algorithm which is applicable to any type of optimization problem (Randeep Kaur et al.,2014) In this paper, a technique is proposed using PSO to find the values of weighting functions to synthesize controller. The proposed PSO technique provides a solution within the short time to calculate the weighting functions for a H-infinity loop shaping controller.

4.1. Weight Selection

The most essential a step in H-infinity loop shaping controller is selection of weighting functions. Roughly speaking, few researchers found that the selection principally influenced by the plant model. Generally, performance and robustness of the weighting functions are selected by trial and error methods. Hence it is most challenging to satisfy all the performance criteria of the robust controller simultaneously. Though there are no specific approaches available for the choice of calculating weighting functions, certain generalization can be adopted from loop shaping procedure (McFarlane. D et al., 1992). The general procedure has the drawback that the number of parameters are to be determined for calculating the weighting functions. The value of the selected weight (stability margin) can be used as a signifying parameter to indicate the quality of the selected weight to satisfy the robust stability criteria. Though, in certain instances, the closed-loop system response of a nominal plant in time domain is not fulfilled but still ε_{opt} is satisfiedIn this article, the efficiency parameters are specified and the optimal weight of W1 is calculated using the PSO. The fitness feature for weight selection is given as *Fitness* = ε_{opt} to meet performance specifications.

4.2. Controller Synthesis

The controller(K) designed through the proposed technique is fixed with a structure and to achieve the optimal stability range, the PSO is applied to determine the optimum parameter p. In the proposed method, the stability margin (ε) is an unique index that determines the stable output of $K\infty$ by

$$K\infty = W_1^{-1} K W_2^{-1} \tag{26}$$

Assume that W1 and W2 are invertible. The objective function is:

Objective function= $\varepsilon = \|T_{zw}\|_{\infty}$

$$=\left\| \begin{bmatrix} I \\ W_1^{-1}K(p) \end{bmatrix} \left(1 - G_s W_1^{-1} K(p)\right)^{-1} [I \;\; G_s] \right\|_{\infty}^{-1} \quad (27)$$

The controller (K) is designed such that the infinity norm is to be minimized from the load disturbances and to maximize the stability margin using PSO methodEvery particle value of fitness or objective function is measured in each iteration of the PSO algorithm. The current iteration value is obtained from the highest fitness value of the particle. The PSO parameters can be modified using (28), (29) and (30) respectively.

$$Q = Q_{max} - \left(\frac{Q_{max} - Q_{min}}{i_{max}}\right) i \quad (28)$$

$$v_{i+1} = Q v_i + \alpha_1[\gamma_{1i}(P_b - p_i) + \alpha_2[\gamma_{2i}(U_b - p_i) \quad (29)$$

$$p_{i+1} = p_i + v_{i+1} \quad (30)$$

where α_1 and α_2 are acceleration coefficients, γ_{1i} and γ_{2i} are any random numbers in (0-1) range The system parameters p are represented as a particle by means of the PSO technique and the fitness can be written as:

Fitness function=

$$\varepsilon = \|T_{zw}\|_{\infty} = \left\| \begin{bmatrix} I \\ W_1^{-1}K(p) \end{bmatrix} \left(1 - G_s W_1^{-1} K(p)\right)^{-1} [I \;\; G_s] \right\|_{\infty}^{-1} \quad (31)$$

The PSO parameters are chosen as in Table 3.

Table 3. PSO parameters

Operating points	Parameters	Value(Min and Max)
Minimum phase	C1	0.01 and 1
	C2	0.1 and 20
	C3	0.0001 and 0.01
	C4	0.001 and 0.3
	Number of particles	20
	Number of Iterations	20
Non-minimum phase	C1	10 and 25
	C2	5 and 17
	C3	0.001 and 0.03
	C4	6 and 20
	Number of particles	20
	Number of Iterations	20

5. DESIGN OF GWO BASED H-infinity LOOP SHAPING CONTROLLER

GreyWolf optimization(GWO) algorithm (S. Mirjalili, et al.,2014, Mirjalili, et al.,2015, S. Mirjalili, et al.,2016) is a latest and popular swarm intelligence based algorithm which is developed in the year 2014 by Mirjalili et al. The GWO algorithm mimics the control structure and the method of shooting grey wolves in the mountains. There are four classes of simulations that are applicable in grey wolf structure as presented in Figure 3(P.B. de Moura Oliveira et al.,2016, A. Madadi.et al 2014).

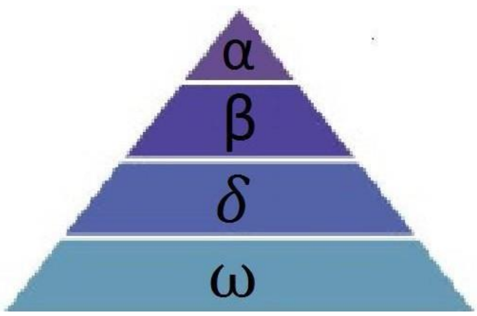

Fig.3. The grey wolf hierarchy

The *Alpha (α)* wolf are the leaders of the entire group which are highly responsible for decision making about various mechanisms like hunting, sleeping place, time to wake, etc. The subordinate of *Alpha (α)* wolves is *Beta (β)* which is at the second rank of hierarchy assist the *Alpha (α)* in decision making for hunting mechanism and other activities (S.K. Verma et al.,2017, D. K. Lal, et al.,2016. P. W. Tsai, et al.,2016) Generally, the solution searches in this algorithm initiates with population of wolves(solutions) that generates in random mannerThese wolves estimate the position of the prey (optimum) through iterative methods during the hunting (optimization) process. Alpha (α) is the most suitable solution preceded by Beta (β) and Delta (δ) as the second and third best solution. The remaining options are of the least value and are called Omega (ω) (S. Shahrazad et al., 2015).

The following equations are suggested to mathematically modulate the encircling behavior.:

$$\vec{D} = |\vec{C}X_P(t) - \vec{X}(t)| \tag{32}$$

$$\vec{X}(t+1) = |\overrightarrow{X_P}(t) - \vec{A}\vec{D}| \tag{33}$$

If t is the current iteration, $\vec{A}$ is a coefficient vector, and $\vec{C}$ is a coefficient variable, $\overrightarrow{X_P}(t)$ is the victim's position variable. X suggests the place vector of the gray wolf. The current iteration vectors, $\vec{A}$ and $\vec{C}$, are calculated as follows:

$$\vec{A} = 2\vec{a}\vec{r}_1 - \vec{a} \tag{34}$$

$$\vec{C} = 2\vec{r}_2 \tag{35}$$

When a include is linearly reduced from 2 to 0 during iterations and r 1 and r 2 are random vectors within the range [0, 1]. The following methods was provided for upgrading the locations of the best search agents.

$$\overrightarrow{D_\alpha} = |\overrightarrow{C_1}X_\alpha - \vec{X}|, \overrightarrow{D_\beta} = |\overrightarrow{C_2}X_\beta - \vec{X}|,$$

$$\overrightarrow{D_\delta} = |\overrightarrow{C_3}X_\delta - \vec{X}| \tag{36}$$

$$\vec{X} = \overrightarrow{X_\alpha} - \overrightarrow{A_1}\vec{D}_\alpha, \vec{X} = \overrightarrow{X_\beta} - \overrightarrow{A_1}\vec{D}_\beta, \quad \vec{X} = \overrightarrow{X_\alpha} - \overrightarrow{A_1}\vec{D}_\delta \tag{37}$$

$$\vec{X}(t+1) = \frac{\overrightarrow{X_1} + \overrightarrow{X_2} + \overrightarrow{X_3}}{3} \tag{38}$$

The GWO algorithm parameters are shown in Table 4.

Table 4. GreyWolf optimizer parameters

Operating points	Minimum Phase	Non-Minimum Phase
Maximum iteration	20	20
Population size	20	25
Number of Decision variables	04	04

6. SIMULATION AND RESULTS:

The step input is given to the QTS system in both minimum and non-minimum phase with a step change at t=500sec and disturbance is added to the system at t=1500sec. After the disturbance again the step-change applied to the system at t=2500 sec. The designed controllers will carry out to track servo and regulatory operations. The MATLAB environment is used to program PSO and GWO algorithms. The algorithms therefore proposed to operate offline for 20 iterations in order to find the minimum value of the cost function that is well known as the objective function. In addition to the optimization process, the optimum GWO algorithm and PSO algorithm provide the best set of weighting functions corresponding to the minimum objective function value. The system is simulated with tuned weights of the H-infinity controller and the results are shown for both minimum and non-minimum phase for the figure (4-11). As seen in Figure 4-11, H-infinity controllers that are tuned using GWO algorithm and PSO algorithm guarantee both regulatory response and servo response. The performances of the proposed algorithms similar. However, as seen from the responses (fig 4-11), the GWO algorithm has a fast response in both minimum and non-minimum phase. Evidently, as can be seen, the GWO algorithm converges faster at the end than the PSO algorithm. In other words, the GWO algorithm provides better strategies for identifying weights relative to the PSO algorithm in terms of set point monitoring, load disturbance rejection and plant instability robustness.

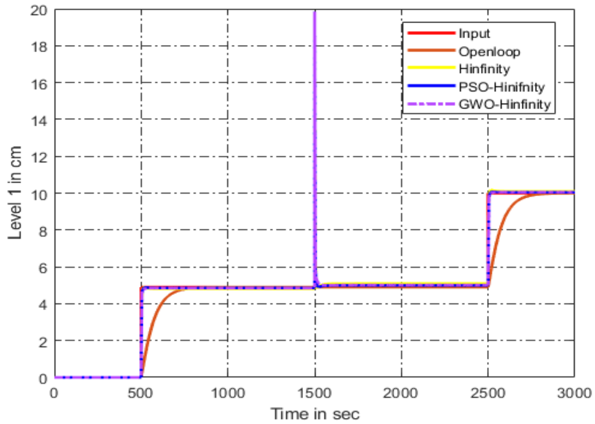

Fig.4. Responses of minimum phase QTS for servo and regulatory operation applied to loop 1

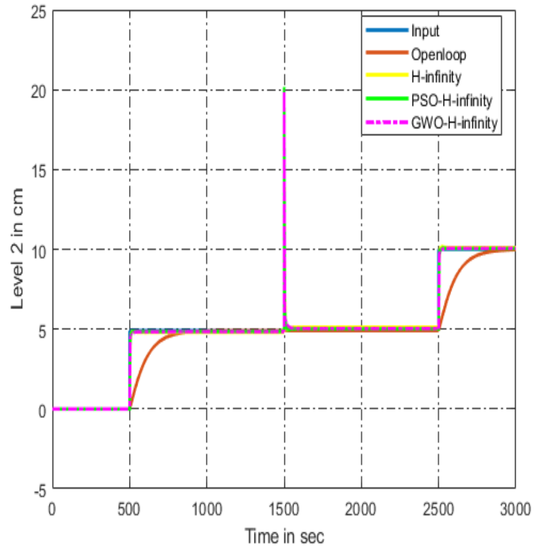

Fig.5. Responses of minimum phase QTS for servo and regulatory operation applied to loop 2

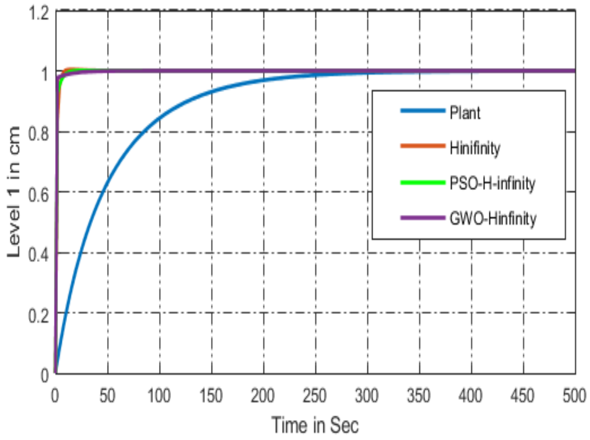

Fig 6 Open loop-shaped response of loop 1 in the minimum phase

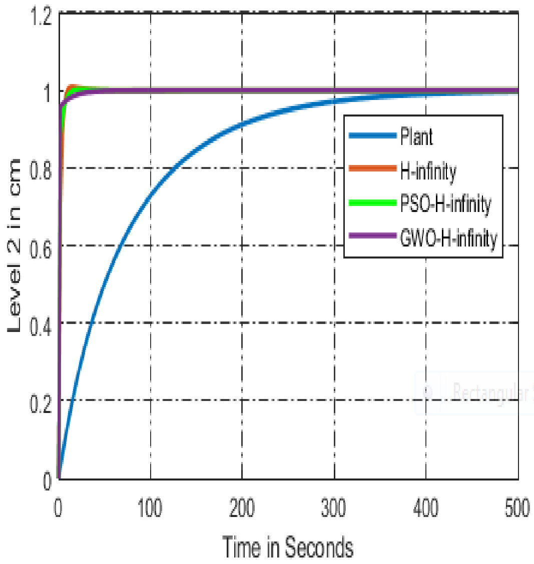

Fig 7 Open loop-shaped response of loop 2 in the minimum phase

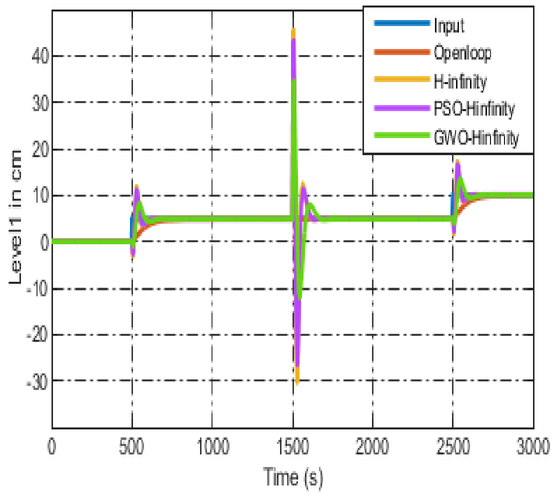

Fig 8 Responses of non-minimum phase QTS for servo and regulatory operation applied to loop 1

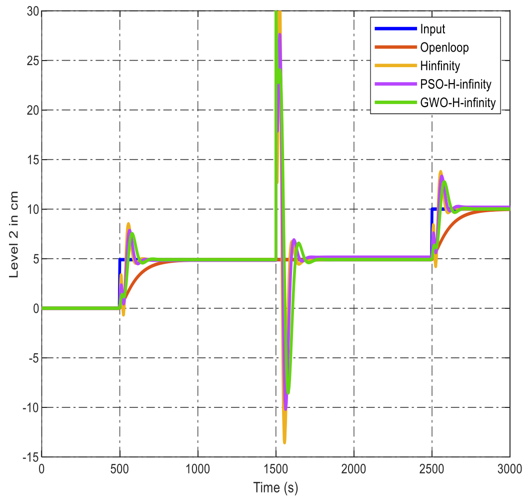

Fig 9 Responses of non-minimum phase QTS for servo and regulatory operation applied to loop 2

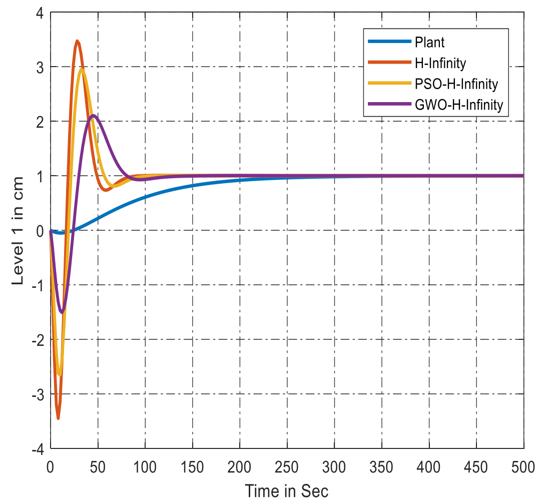

Fig 10 Open loop-shaped response of loop 1 in non-Minimum phase

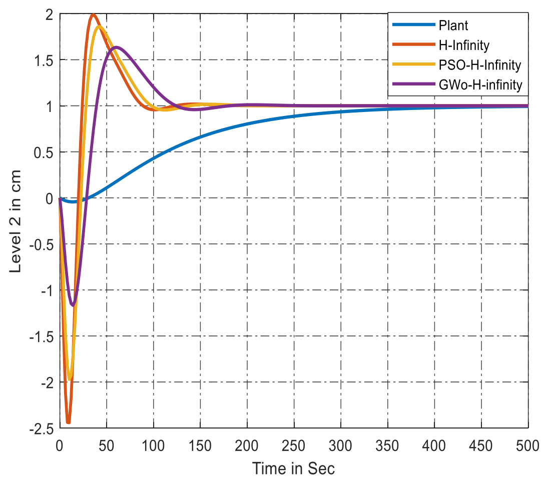

Fig 11 Open loop-shaped response of loop 2 in non-minimum phase

The performance of the designed H-infinity controller, PSO tuned H-infinity controller and GWO tuned H-infinity controller are compared and tabulated in table 5.

Table 5 Comparison of controller performance

Operating points		Parameters	Controller		
			HILSC	PSO tuned HILSC	GWO tuned HILSC
Minimum phase	Level 1	Gamma	1.422	1.4	1.338
		Integral square error(ISE)	550.5	244.7	51.84
		RiseTime(in sec)	4.093	1.935	1.292
		Overshoot(%)	190.38	96.91	91.51
	Level 2	Gamma	1.422	1.4	1.338
		Integral square error(ISE)	850.9	414.4	86.9
		Rise Time (in sec)	4.67	2.680	2.294
		Overshoot (%)	190.38	96.91	91.51
Non-minimum phase	Level 1	Gamma	30	26.19	14.65
		Integral square error (ISE)	5668	746.7	487
		Rise Time (in sec)	3.27	2.945	9.406
		Overshoot (%)	143.019	229.892	99.98
	Level 12	Gamma	30	26.19	14.65
		Integral square error (ISE)	459	428.6	467.9
		Rise time Time (in sec)	4.403	20.405	30.434
		Overshoot (%)	227.204	221.992	105.74

7. CONCLUSIONS:

In this paper, H-infinity loop shaping controller(HILSC) for the quadruple tank system are designed to ensure a multi-loop control in both minimum and non-minimum phase using the classical method, PSO algorithm and GWO algorithm and simulations are examined The significance of this work is that for the first time, GWO is used as a variety method for a quadruple tank system to adjust H-infinity controllers. In comparison, to analyze the output of GWO, it is contrasted with a common search algorithm such as PSO. The GWO algorithm was effectively used for the configuration of H-infinity controllers. The suggested algorithms are used to determine the optimum weights for optimal controller output in the quadruple tank system. The findings of the simulation reveal that the GWO algorithm is faster and more effective than the PSO algorithm in the global and local optimization search. In this study, the GWO algorithm is the best that demonstrates the satisfactory performance of the PSO algorithm. Nevertheless, the tuned weights of the H-infinity controllers performed servo and regulatory control of the system productively with all the proposed algorithms. It is anticipated that this method will contribute more to the quadruple tank system that works about most of the multivariable systems especially with the non-minimum phase. The time-domain specifications analysis of the designed controller has produced better results. Hence, the proposed controller can be applied to any multivariable system with control challenges.

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