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Multi-Model Based Robust LPV-H∞ Control and Observation Design of non Linear Multivariable Three Tank System

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Abstract: This paper deals with a design a robust multi-variable control technique based on a multi-model (MM) Linear Parameter Varying (LPV) approach coupled with «H∞ » synthesis. The developed technique is applied to a three-tank non linear system (NL). The first part concerns the modeling of the three-tank system by using two approaches (MM based on linearization (MM-L) around several equilibrium points and MM quasi Linear Parameters Varying (MM-qLPV) polytopic approach. The second part is devoted to the design of two control laws: a multivariable polytopic state feedback control and a robust multivariable MM-H∞ control by dynamic output feedback (using MM polytopic formulation and Linear Matrix Inequalities (LMI)). Indeed, these control laws are based on the development of several local controllers (one controller per local model). Then, with a convex polytopic formulation, the MM approach combines these controllers to form a global controller that stabilizes the complete non linear system. Besides, a multi-observer (MM based observer) is used to estimate the immeasurable states. Finally, the two synthesized global controllers are applied to the water level regulation of the previously modelled three-tank system. Furthermore, a comparison between the robust performances of these two controllers is made in nominal and disturbed modes.

Keywords: Robust control, Multi model-H∞ synthesis, LMI, quasi-LPV modeling, Multi-observers, state feedback.

1. INTRODUCTION

In automatic control the synthesis of a control law is generally based on a simplified nominal model that does not take into account the complexity of the system. In reality all systems and physical phenomena have non-linear or time-varying behaviors. To study these systems in different contexts (identification, control and diagnosis) the use of a non-linear model is more difficult and more delicate than the linear one. Several researchers have therefore adapted some linear techniques to apply them to non-linear systems. Among the modeling techniques allowing the representation of a non-linear system with the use of techniques adapted to linear models one can cite the multi-model (MM) approach, which is based on multiple Linear Time Invariants (LTI) models. This approach has been developed for several years as stated in (Murray et al., 1997). It is a convex polytopic representation that can be obtained either directly by a transformation of the non-linear mathematical model into a set of refined models in the state (see Rodrigues, 2005), by linearization of the non linear model around different operating points or from black box models based on the system's inputs and outputs data. Therefore, a description of the overall model is achieved through an interpolation of different local models and uses the weighting functions associated with each of them. This approach has produced interesting results for the observation, diagnosis and control of ordinary non-linear systems. Several categories of MM exist in the literature, such as linear systems with time-varying (LTV) or parameters varying (LPV) (Zerar et al., 2009) or quasi-LPV (q-LPV) systems, still known as Takagi-Sugeno (T-S) systems (Takagi et al., 1985). The MM approach has a universal approximation property of systems and has the advantage of being able to accurately represent a non-linear model on a compact of state space representation. Besides, MM approach has been used for several control problems. Feedback stabilization has been addressed for several types of T-S models and models with bounded parametric uncertainties as in (Chen et al., 2000) or delayed systems in (Cao et al., 2001). In fact, the most linear system control techniques are based on accurate knowledge of the mathematical model. In reality, it is almost impossible to accurately describe the behavior of real process. The basic idea is that it would be more fruitful to consider a process model as the combination of nominal models with uncertainties (structured or unstructured). In this case, the control issue must be addressed in such a way as to ensure robustness in relation to these uncertainties. Conventional commands such as PID or standard feedback control are limited and not capable (in some cases) to ensure optimal system's operation when various disturbances (internal or external) appear. This results in a deterioration of process performances, and may even induce system instability. It is therefore necessary to use robust advanced control techniques, capable of taking into account system uncertainties, disturbances, and ensuring a good process behavior. There are many methods for robust controller's synthesis. The focus in this paper will be on an LPV-H∞ based synthesis technique, where the designed controller must satisfy stability and robust performances against parametric uncertainties and external disturbances. The three tank system is considered as an application to validate the proposed techniques. Since a long time, the three tank system is considered as a benchmark and has attracted researchers to develop and test several control strategies. In (Stoustrup et al., 2000) a combined feedback control and fault detection and identification is applied to three tank system. The design problem is formulated as an H∞ standard design problem, but only one linearized model is considered. In (Galindo, 2005), based on a stable pre-compensator and its dual post-compensator, a mixed sensitivity H∞ control methodology is tested on a Three-Tank-System but it did not consider the whole system but it has also taken one linearized model (obtained around one operating point). In (Qingfeng et al., 2006) a three tank system with data packets loss is modelled as a discrete-time Markov jumping linear system and controlled by H∞ control approach, but only two linearized models have been considered (one linearized model when measurement packet is transmitted and another one when the packet is lost). In (Zhenping et al., 2012) a multi-model predictive PID cascade control strategy was presented for level control of three-tank system. A fuzzy controller has been designed in (Mastacan et al., 2013), a bees algorithm has been used in a model predictive control (MPC) in order to control three-tank systems in (Sarailooa et al., 2015). A constrained PD controller design with exact linearization in (Bistak, 2015). RBF-ARX model-based MPC in (Zhou et al., 2015). Interval type-II Fuzzy Logic controller is applied to three tank system in (Hituraj et al., 2016). A disturbance observer based backstepping controller in (Zi-Jiang et al., 2017). Adaptive based control in (Elmira et al., 2017). Multiple linearized approach based on LQR design in (Sathishkumar et al., 2017). Two degree of freedom model reference control with dynamical principle model feedforward control extended by a nonlinear disturbance observer in (Bistak, 2018). A feedforward controller with a nonlinear model predictive controller in (Shuyou et al., 2018). A fuzzy sliding mode and linear matrix inequality approach in (Mellouli et al., 2018). A finite-time disturbance observer-based non linear control combined with dynamic surface control in (Zi-Jiang et al., 2019). From all the mentioned literature and to the author's knowledge there are no results on the application of MM-H∞ control technique to the nonlinear three tank system. Motivated by this, the main purpose of this paper is the development and the application of a MM-H∞ control/observation strategy to an uncertain non linear multivariable system which is a three tanks system. Firstly, three modeling techniques devoted to non-linear systems (Linear Parameter Varying (LPV), Multi-Model (MM) technique by linearization around different operating points, MM based convex polytopic transformation) have been adopted and applied to the three tanks system. Then, based on linear matrix inequality (LMI) technique, and by using the developed Multi Model, a MM-H∞ observer/controller is synthesized and compared with a multivariable polytopic state feedback controller.

The paper is organized as follows: after this introduction, three tank system description and its non linear modeling are provided in Section 2. Section 3 presents a quasi-LPV model of the three tank system and shows a comparison between this model and the non linear model. Section 4 gives a multi model reformulation (MM-qLPV) of the q-LPV model previously developed. The linearization approach of the non linear model is also presented in this section and compared with the MM-qLPV approach. Section 5 states the MM based Multi obsever design problem where the synthesis conditions are expressed in terms of strict LMI. Then, the MM-qLPV model is used in section 6 for polytopic state feedback control design and in section 7 for MM-H∞ control synthesis. A comparison between these two developed controllers (state feedback and MM-H∞ is tackled in Section 8. The paper ends with a conclusion.

Notations: M^T denotes the conjugate transpose of matrix M and $\|M\|_\infty$ stands for the ∞-norm of matrix M induced from the Euclidean vector-norm. M^{-1} is the inverse of square matrix M and M^H is the Hermitian matrix defined by $M + M^T$. O and I denote the zero matrix and the identity matrix of appropriate dimension respectively. Finally, Matrix inequalities are considered in the sense of Löwner, i.e. <0 (resp. ≤ 0) stands for (semi-)negative definite and >0 (resp ≥ 0) stands for (semi-)positive definite.

2. THREE-TANK SYSTEM DESCRIPTION

Three-tank system is a multi-input and multi-output (MIMO) non-linear hydraulic system, consisting of three tanks T_1, T_2 and T_3, one tarpaulin B_0 and two pumps P_1 and P_2. Each tank is connected to the tarpaulin by a duct of section *Sn* whose flow is modulated by a manual valve. In addition, two ducts of the same sections, whose flow is modulated by a valve, allow to connect T_1, T_3 and T_2 tanks respectively. The pumps operate one-way and are controlled in flow. Water levels in the tanks, noted h_1, h_2 are measured by sensors placed on the tanks, and the h_3 level is immeasurable.

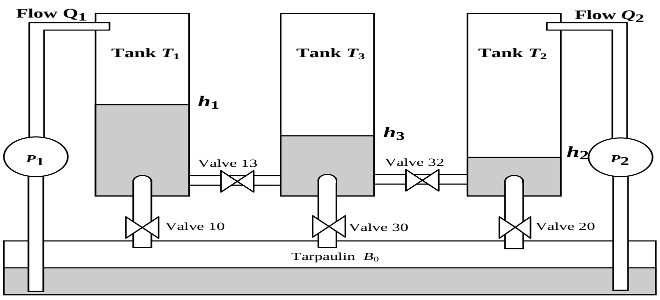


Fig. 1. Three-tank system schematic diagram.

This system is represented by the following equations (as in (AMIRA, 1996):

$$\begin{cases} S_c \dfrac{dh_1}{dt} = -Q_{10}(h_1) - Q_{13}(h_1, h_3) + Q_1 \\ S_c \dfrac{dh_2}{dt} = -Q_{20}(h_2) + Q_{32}(h_2, h_3) + Q_2 \\ S_c \dfrac{dh_3}{dt} = -Q_{30}(h_3) - Q_{32}(h_2, h_3) + Q_{13}(h_1 h_3) \end{cases} \quad (1)$$

Where:

$$Q_{i0} = az_{i0} S_n \sqrt{2gh_i} = a_{i0}\sqrt{h_i} \quad (2)$$

$$Q_{ij} = az_{ij} S_n sign(h_i - h_j)\sqrt{2g|h_i - h_j|} = a_{ij} sign(h_i - h_j)\sqrt{|h_i - h_j|} \quad (3)$$

S_c : section of a tank,

az_{ij}: valve coefficient reflecting the flow rate of the duct connecting the element *i* to the element *j* via the valve *ij*.

Q_{i0} : flow of the tank T_i into the tarpaulin B_0,

Q_{ij} : flow from the tank T_i to the tank T_j.

The state space representation of this system is given by:

$$\begin{cases} \begin{bmatrix} \dot{h_1} \\ \dot{h_2} \\ \dot{h_3} \end{bmatrix} = \frac{1}{S_c}\begin{bmatrix} -Q_{10}(h_1) - Q_{13}(h_1,h_3) \\ -Q_{20}(h_2) + Q_{32}(h_2,h_3) \\ -Q_{30}(h_3) - Q_{32}(h_2,h_3) + Q_{13}(h_1,h_3) \end{bmatrix} + \frac{1}{S_c}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \\ y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \end{cases} \tag{3}$$

3. QUASI LINEAR PARAMETERS VARYING MODEL (qLPV MODEL)

Its principle is based on a linear representation whose parameters vary over the time according to one or more external or internal variables, called sequencing or premise variables (see Chouaba, 2012; Scherer, 2001). This representation is not unique, and to write the system of the three tanks in equivalent qLPV form; one uses a weighted sum of the states $x_i(t)$ thanks to the real scalars λ_i and γ_i $(i = 1,.., n)$. The choice of these scalars gives the possibility to highlight several LPV forms. For λ_i, γ_i and $x_i(t)$ not null, the weighted sums, $\sum_{i=1}^{3}\lambda_i x_i\,(t)$ and $\sum_{i=1}^{3}\gamma_i x_i\,(t)$ are not null. By using these sums, the system state space representation can be written as follows:

$$\dot{x}_1 = -\frac{W_{13}}{\sum_{i=1}^{3}\lambda_i x_i(t)}\sqrt{|x_1 - x_3|}\sum_{i=1}^{3}\lambda_i x_i\,(t) + \frac{Q_1}{Sc} \tag{5}$$

$$\dot{x}_2 = -\frac{W_{32}}{\sum_{i=1}^{3}\gamma_i x_i(t)}\sqrt{|x_3 - x_2|}\sum_{i=1}^{3}\gamma_i x_i(t) - \frac{a_{20}}{Sc}\sqrt{x_2} + \frac{Q_2}{Sc} \tag{4}$$

$$\dot{x}_3 = \frac{W_{13}}{\sum_{i=1}^{3}\lambda_i x_i(t)}\sqrt{|x_1 - x_3|}\sum_{i=1}^{3}\lambda_i x_i(t) - \frac{W_{32}}{\sum_{i=1}^{3}\gamma_i x_i(t)}\sqrt{|x_3 - x_2|}\sum_{i=1}^{3}\gamma_i x_i(t) \tag{7}$$

With : $W_{13} = \frac{a_{13} sing(x_1 - x_3)}{Sc}$; $W_{32} = \frac{a_{32} sing(x_3 - x_2)}{Sc}$

The premise variables are defined as follows:

$$z_1 = \frac{W_{13}}{\sum_{i=1}^{3}\lambda_i x_i(t)}\sqrt{|x_1 - x_3|} \tag{5}$$

$$z_2 = \frac{a_{20}}{Sc}\sqrt{\frac{1}{x_2}} \tag{6}$$

$$z_3 = \frac{W_{32}}{\sum_{i=1}^{3}\gamma_i x_i(t)}\sqrt{|x_3 - x_2|} \tag{7}$$

Then, the system (4) takes the following qLPV form:

$$\begin{cases} \dot{x}(t) = A(z_1, z_2, z_3)x(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{8}$$

Where the matrices $A(z_1, z_2, z_3)$, B and C are defined as follows:

$$A(z_1, z_2, z_3) = \begin{bmatrix} -\lambda_1 z_1 & -\lambda_2 z_1 & -\lambda_3 z_1 \\ \gamma_1 z_3 & \gamma_2 z_3 - z_2 & \gamma_3 z_3 \\ \lambda_1 z_1 - \gamma_1 z_3 & \lambda_2 z_1 - \gamma_2 z_3 & \lambda_3 z_1 - \gamma_3 z_3 \end{bmatrix} \tag{12}$$

$$B = \begin{bmatrix} \frac{1}{Sc} & 0 \\ 0 & \frac{1}{Sc} \\ 0 & 0 \end{bmatrix};$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

Where the premise variables z_1, z_2 and z_3 are the external entries of the matrix A.

The following paragraph summarizes this MM approach and explains how to choose the scalars λ_i and γ_i $(i = 1,.., n)$:

1- Based on the non linear model (1 or 4) and by multiplying and dividing each state equation by $\sum_{i=1}^{3}\lambda_i x_i\,(t)$ and/or $\sum_{i=1}^{3}\gamma_i x_i(t)$, the equations (5, 6 and 7) are obtained.

2- In order to remove the non linear part from these equations (5, 6 and 7), the premise variables z_1, z_2 and z_3 given in equations (8, 9 and 10) are introduced and variable changes are performed. The quasi LPV model structure (11, 12) is then obtained.

3- From mathematical point of view, the model structure (11, 12) is only but a reformulation of the non linear model (1 or 4). The choice of the scalars $\gamma_i$and λ_i gives the possibility to highlight several LPV forms. Thus, the quasi-LPV form (11, 12) is not unique and for each quasi-LPV form there exist a particular premise variable set. Choosing a quasi LPV form is equivalent to choose the premise variable set which directly related to the choice of the scalars $\gamma_i$and λ_i. In fact, this is a degree of freedom that should be used to ease the controllability, the observability and the stability analysis studies. Besides, this degree of freedom can also be used, to minimize the norm of the obtained sub-models i.e. to obtain sub-models with small eigenvalues, which leads to a control law (or observer gain) with small energy.

The simulation result illustrated by fig. 2 shows a comparison between the developed LPV model (11) and the NL model (4). The parameters used for the simulation are: $\lambda_1 = -0.7$, $\lambda_2 = 5$, $\lambda_3 = 17$, $\gamma_1 = 0$, $\gamma_2 = -3$, $\gamma_3 = -1$ and the initial conditions $x_0 = [0.05\ 0.02\ 0.01]$ meter. The control inputs used to excite the system are: a sine wave form for the flow Q_1, and a step form for the flow Q_2. From Fig. 2, it is clear that the LPV model and the non linear model have exactly the same behavior.

Remark: Up to now, it is noted that all possible values of λ_i and γ_i $(i = 1,.., n)$ excluding the degenerated cases $\lambda_1=\lambda_2=\lambda_3=0$ and/or $\gamma_1 = \gamma_2 = \gamma_3 = 0$ give the same simulation results. Besides, all the models (with different λ_i and γ_i) are analytically equivalent with the initial nonlinear form of the three-tank model (1). However, the parameters values used in this simulation will be justified in the next sections.

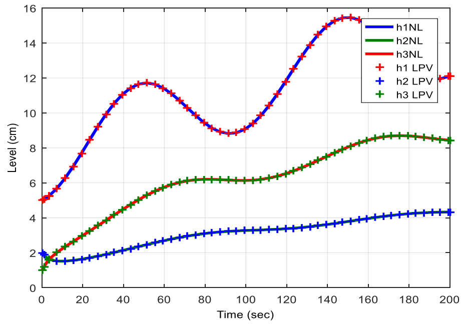


Fig. 2. Comparison between non linear (NL) and LPV models.

4. SYSTEM MODELING BY USING MULTI-MODEL (MM) FORMULATION

Based on MM formulation, two models have been developed by using two different approaches:

a) By linearization around several operating points,

b) Quasi LPV polytopic approach (MM-qLPV).

4.1. MM by Linearization Approach

This approach is based on the linearization of the non-linear mathematical model of the physical process around different operating points (wisely chosen). Let us consider the following non-linear system:

$$\dot{x}(t) = f(x(t), u(t)) \tag{9}$$

The linearization of the system (13) around an arbitrary operating point *(x_{i0}, u_{i0})* is given by:

$$\dot{x}(t) = A_i(x(t) - x_{i0}) + B_i(u(t) - u_{i0}) + f(x_{i0}, u_{i0}) \tag{10}$$

that can be rewritten in the form of:

$$\dot{x}(t) = A_i x(t) + B_i u(t) + d_i \tag{11}$$

with:

$A_i = \frac{\partial f(x,u)}{\partial x}\Big|_{u = u_{i0}}^{x = x_{i0}}$, $B_i = \frac{\partial f(x,u)}{\partial u}\Big|_{u = u_{i0}}^{x = x_{i0}}$, $d_i = f(x_{i0}, u_{i0}) - A_i x_{i0} - B_i u_{i0}$

For the three tank system, the different coordinates of the operating points are obtained by the resolution of the following equations:

$$\begin{cases} -Q_{10}(h_1) - Q_{13}(h_1, h_3) + Q_1 = 0 \\ -Q_{20}(h_2) + Q_{32}(h_2, h_3) + Q_2 = 0 \\ -Q_{30}(h_3) - Q_{32}(h_2, h_3) + Q_{13}(h_1, h_3) = 0 \end{cases} \tag{12}$$

The numerical values of the obtained operating points (x_{10}, x_{20}, x_{30}, u_{10}, u_{20})=(h_{10}, h_{20}, h_{30}, Q_{1e}, Q_{2e}) are summarized in Table 1. For each operating point, a local model is built. Therefore, the matrices of the multi-model are obtained as follows:

$A_i = \frac{\partial f(h,Q)}{\partial h}\Big|_{u = Q_{ie}}^{h = h_{i0}}$, $B_i = \frac{\partial f(h,Q)}{\partial Q}\Big|_{Q = Q_{ie}}^{h = h_{i0}}$, $d_i = f(h_{i0}, Q_{ie}) - A_i h_{i0} - B_i Q_{ie}$

Table 1. Different operation points.

i	Q_{1e}	Q_{2e}	h_{10}	h_{20}	h_{30}
1	1.10^{-5}	1.10^{-5}	0.0330	0.0166	0.0248
2	1.10^{-5}	3.10^{-5}	0.0829	0.0666	0.0747
3	1.10^{-5}	5.10^{-5}	0.1661	0.1498	0.1579
4	3.10^{-5}	1.10^{-5}	0.2134	0.0666	0.1400
5	3.10^{-5}	3.10^{-5}	0.2966	0.1498	0.2232
6	3.10^{-5}	5.10^{-5}	0.4131	0.2663	0.3397
7	5.10^{-5}	1.10^{-5}	0.5575	0.1498	0.3537

Where:

$$A_i = \begin{bmatrix} -\frac{a_{13}}{2ScQ_{1ei}} & 0 & \frac{a_{13}^2}{2ScQ_{1ei}} \\ 0 & -\left(\frac{a_{32}^2}{2ScQ_{1ei}} + \frac{a_{20}^2}{2Sc(Q_{1ei} + Q_{2ei})}\right) & \frac{a_{32}^2}{2ScQ_{1ei}} \\ \frac{a_{13}^2}{2ScQ_{1ei}} & \frac{a_{32}^2}{2ScQ_{1ei}} & -\frac{(a_{32}^2 + a_{13}^2)}{2ScQ_{1ei}} \end{bmatrix},$$

$$B_i = \frac{1}{Sc}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{13}$$

The previously obtained local models are implemented in Matlab/Simulink. A simple switching algorithm is also implemented in order to switch between these models depending on the operating points. The obtained simulation results are illustrated in (Fig. 3) and (Fig. 4). In (Fig. 3) a comparison between the NL model and the MM model is performed. The sequencing variable used to switch between the models is shown in (Fig. 4). The (Fig. 3) shows that there are a transient (picks) at each commutation instants. These picks could produce actuator saturation and causes a deterioration of the system performances. This problem will be treated by the MM-LPV polytopic approach presented in the next subsection.

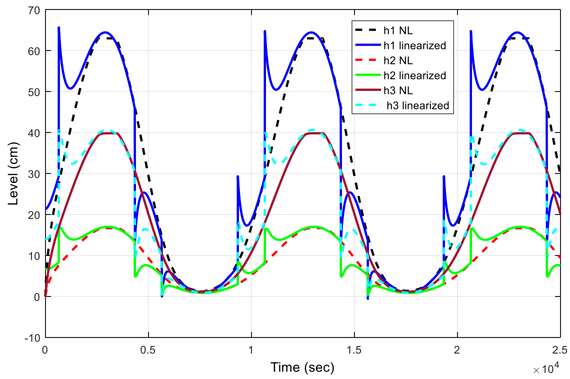


Fig. 3. Comparison between NL and linearized models.

4.2. MM-LPV polytopic approach

A MM-qLPV polytopic form is developed by using the polytopic convex transformation (see Kiss et al., (2009). In fact, the premise variables z_j given by (8), (9) and (10) can be written as follows:

$$z_j = F_{j,1}(z_j(\rho(x,u))z_{j,1} + F_{j,2}(z_j(\rho(x,u))\, z_{j,2} \tag{14}$$

Where the scalars $z_{j,1}$ and $z_{j,2}$ are defined by:

$$z_{j,1} = \max_{x,u}(z_j(\rho(x,u)) \tag{15}$$

$$z_{j,2} = \min_{x,u}(z_j(\rho(x,u)) \tag{16}$$

and the functions $F_{j,1}$, $F_{j,2}$ are given by:

$$F_{j,1}(z_j(\rho(x,u))) = \frac{z_j(\rho(x,u)) - z_{j,2}}{z_{j,1} - z_{j,2}} \tag{17}$$

$$F_{j,2}\left(z_j(\rho(x,u))\right) = \frac{z_{j,1} - z_j(\rho(x,u))}{z_{j,1} - z_{j,2}} \tag{18}$$

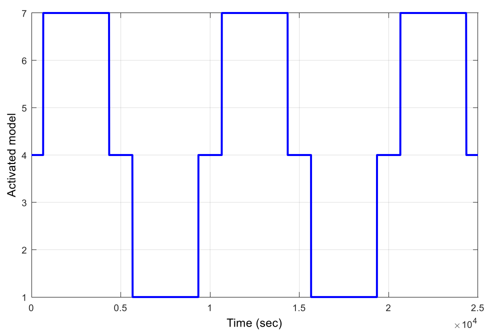


Fig. 4. Sequencing between different sub-models.

Thus, the MM is composed of $r = 2^3$ sub-models. The weighting functions are calculated as follows:

$$\mu_i(x) = \prod_{j=1}^{3} F_{j,\sigma_i^j}\left(z_j(\rho(x,u))\right) \quad (19)$$
$$= F_{1,\sigma_i^1}(z_1)\, F_{2,\sigma_i^2}(z_2)\, F_{3,\sigma_i^3}(z_3)$$

The variables $F_{i,j}$ for each sub-model are summarized in Table 2.

Table 2. Construction of MM with three premise variables.

Model i	Partitions						σ_i
	z_1		z_2		z_3		
	$F_{1,1}$	$F_{1,2}$	$F_{2,1}$	$F_{2,2}$	$F_{3,1}$	$F_{3,2}$	
1	1	0	1	0	1	0	(1,1,1)
2	1	0	1	0	0	1	(1,1,2)
3	1	0	0	1	1	0	(1,2,1)
4	1	0	0	1	0	1	(1,2,2)
5	0	1	1	0	1	0	(2,1,1)
6	0	1	1	0	0	1	(2,1,2)
7	0	1	0	1	1	0	(2,2,1)
8	0	1	0	1	0	1	(2,2,2)

Where the index vector $\sigma_i = (\sigma_{i,1},.., \sigma_{i,p})$ containing values 1or 2 provides the value of the index i representing the number of the obtained sub-model. The constant matrices of each sub-model are given by:

$$A_i = A\left(z_{1,\sigma_i^1}, z_{2,\sigma_i^2}, z_{3,\sigma_i^3}\right), B_i = B, C_i = C,\ \ i = 1, \dots, 8. \quad (24)$$

Finally, the NL system (4) is rewritten as a MM:

$$\begin{cases} \dot{x}_i(t) = \sum_{i=1}^{8} \mu_i\left(x(t)\right)[A_i x(t) + Bu(t)] \\ y(t) = Cx(t) \end{cases} \quad (20)$$

Note that the weighting functions μ_i and the matrices A_i depend on the scalars λ_i and γ_i $(i = 1, 2, 3)$. As said before, for different choices of these scalars, different MM will be obtained (except the case where $\lambda_1 = \lambda_2 = \lambda_3 = 0$ and/or $\gamma_1 = \gamma_2 = \gamma_3 = 0$ which is not possible because it gives a division by zero in (8) and (10)).

4.3. *Choice of MM-LPV structure and parameters*

For the choice of the MM structure it is necessary to take into account the conditions to be respected (for example the observability and the controllability of the system) by a good choice of the real scalars λ_i and γ_i $(i = 1, 2, 3)$. This choice is key point in order to be able to synthesize both observer and controller for the three tanks system. The following points explain how to choose the scalars λ_i and γ_i.

1- First of all, the controllability and observability of each sub-model is necessary to ensure the controllability and observability of the global system, represented in a multiple model form. Thus, the scalars producing sub-models which are not controllable or observable will not be considered.

2- One eliminates all the quasi-LPV forms for which the matrices have null columns and or rows. For example: if the parameters $\gamma_3 = 0$ and $\lambda_3 = 0$ are chosen, the observability is not respected as shown below.

a- The system observability matrix $\mathcal{O}$ is given by :

$$\mathcal{O} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\lambda_1 z_1 & -\lambda_2 z_1 & -\lambda_3 z_1 \\ \gamma_1 z_3 & \gamma_2 z_3 - z_2 & \gamma_3 z_3 \\ (*)_{51} & (*)_{52} & (*)_{53} \\ (*)_{61} & (*)_{62} & (*)_{63} \end{bmatrix}$$

Where $(*)_{ij}$ represent the following terms:

$$(*)_{51} = \lambda_1 z_1^2(\lambda_1 - \lambda_3) + \gamma_1 z_1 z_3(\lambda_3 - \lambda_2)$$
$$(*)_{52} = \lambda_2 z_1^2(\lambda_1 - \lambda_3) + \gamma_2 z_1 z_3(\lambda_3 - \lambda_2) + \lambda_2 z_1 z_2$$
$$(*)_{53} = \lambda_3 z_1^2(\lambda_1 - \lambda_3) + \gamma_3 z_1 z_3(\lambda_3 - \lambda_2)$$
$$(*)_{61} = \lambda_1 z_1 z_3(\gamma_3 - \gamma_1) - \gamma_1 z_3[z_2 + z_3(\gamma_3 - \gamma_2)]$$
$$(*)_{62} = (\gamma_2 z_3 - z_2)^2 + \lambda_2 z_1 z_3(\gamma_3 - \gamma_1) - \gamma_2\gamma_3 z_3^2$$
$$(*)_{63} = -\lambda_3\gamma_1 z_1 z_3 + \gamma_3 z_3[(\gamma_2 - \gamma_3)z_3 - z_2 + \lambda_3 z_1]$$

For $\gamma_3 = 0$ and $\lambda_3 = 0$, the terms $(*)_{53} = 0$ and $(*)_{63}=0$, then the third column of the observability matrix $\mathcal{O}$ is null and rank$(\mathcal{O})$ =2 therefore the system is not observable. Then, one can conclude that all the sub-models that can be obtained from (11, 12) are observables for $\gamma_3 \neq 0$ and $\lambda_3 \neq 0$.

b- The system controllability matrix Q is given by:

$$Q = [B \quad AB \dots A^{n-1}B] = [B \quad AB \quad A^2B]$$
$$= 1/Sc \begin{bmatrix} 1 & 0 & -\lambda_1 z_1 & -\lambda_2 z_1 & * & * \\ 0 & 1 & \gamma_1 z_3 & \gamma_2 z_3 - z_2 & * & * \\ 0 & 0 & \lambda_1 z_1 - \gamma_1 z_3 & \lambda_2 z_1 - \gamma_2 z_3 & * & * \end{bmatrix},$$

Like the observability, the same analysis is performed for the controllability matrix Q. As an example, if one take $\gamma_1 = 0$, and taking into account that the degenerate cases ($x_1 = x_3$, $x_2 = x_3$ and $x_1 = x_2 = x_3 = 0$) are excluded, the maximum and the minimum of the premise variables are always non null ($z_1 \neq 0$), one can conclude from the third column of Q that all the sub-models are controllable if $\lambda_1 \neq 0$. ($rank(Q)=3$ for $\lambda_1 \neq 0$). Similar studies can be realized for the other possible cases.

3- Secondly, it is noted that the degree of freedom given by the scalars γ_i and λ_i is also used to minimize the energy of the controller output u trough the norm-2 minimization of the eigenvalues of the sub-models matrices A_i $(i=1,..,8)$ of (25). In fact, it is clear that when a system has a big (positive) eigenvalues, this need a big effort (energy) from the controller to stabilize it. So a justified good choice of γ_i and λ_i is the one who leads to a set of models (sub-models whose matrics A_i have small eigenvalues.

To summarize, the scalars γ_i and λ_i are chosen to ensure submodels observability and controllability and minimal norm-2 of the eigenvalues of the sub-models matrices A_i of the MM given by (25).

5. MULTI-OBSERVER DESIGN

As mentioned before, a synthesis of an observer is necessary in order to estimate the immeasurable state of the system.

5.1. *Observervability condition*

To build an observer based on the developed MM, the observability of each sub-model is necessary to ensure the observability of the global MM. The following geometric condition is used (Kiss, 2010; Chi-Tsong Chen,1999):

$$rang(O_i) = rang\begin{bmatrix} C_i \\ C_iA_i \\ \vdots \\ C_iA_i^{n-1}\end{bmatrix} = n \quad \forall i = 1,\ldots,8 \tag{21}$$

Where n is the system order. The matrices A_i $(i = 1, \ldots, 8)$ have the same structure as the matrix A. The difference is that each variable z_1, z_2 and z_3 takes its maximum or minimum. Assuming that degenerated cases are excluded ($x_1 = x_3$, $x_2 = x_3$ and $x_1 = x_2 = x_3 = 0$), where the maximum and minimum of the premise variables are nonzero. Therefore, the eight observability conditions (26) have to be verified.

5.2. Synthesis of a MM observer

The method used here for the synthesis of an observer is based on the MM (18). Consider the matrices A_0 and A_i defined by:

$$A_0 = \tfrac{1}{r}\textstyle\sum_{i=1}^{r} A_i,\ \bar{A}_i = A_i - A_0,\ r=8 \tag{27}$$

By substituting A_0 and $\bar{A}_i$ in the state equation of the MM (25), the Multi-observer used for the three-tank system has the following form (as in Kiss, 2010):

$$\begin{cases} \dot{\hat{x}} = A_0\hat{x} + \sum_{i=1}^{r} \mu_i(\hat{x})[\bar{A}_i\hat{x} + Bu + L(y-\hat{y})] \\ \hat{y} = C\,\hat{x} \end{cases} \tag{22}$$

Where L is the observer gain (a matrix to be determined). The state estimation error is given by:

$$e(t) = x(t) - \hat{x}(t) \tag{23}$$

Its dynamic is:

$$\dot{e}(t) = (A_0 - LC)e(t) + \Delta(x,\hat{x}) \tag{24}$$

where:

$$\Delta(x,\hat{x}) = \sum_{i=1}^{r} \bar{A}_i(\mu_i(x)x - \mu_i(\hat{x})\hat{x} + B(\mu_i(x) - \mu_i(\hat{x})u) \tag{25}$$

According to the convergence theorem of the estimation error presented in (Ichalal et al., 2008) (Lemma 1), the state estimation error between the MM (25) and the observer (28) asymptotically converges to zero, if there exist matrices $P = P^T > 0$, $Q = Q^T > 0$ and K such that the following LMI (Boyd, S. et al. (1994)) conditions are met:

$$\begin{bmatrix} A_0^TP + PA_0 - C^TK^T - KC + \psi^2Q & P \\ P & -Q\end{bmatrix} < 0 \tag{32}$$

The observer gain (28) is given by:

$$L = P^{-1}K \tag{33}$$

It should be noted that the number of LMIs does not depend on the number of sub-models. It only depends on A_0, the average matrix of the sub-models matrices A_i. Nevertheless, A_0 and A_i share the same structure, so the choice of the MM is important in determining the gain L. For the simulation, the parameters $\lambda_1 = -0.7$, $\lambda_2 = 5$, $\lambda_3 = 17$, $\gamma_1 = 0$, $\gamma_2 = -3$, $\gamma_3 = -1$ and $\psi = 0.01$ have been used. These parameters were chosen to comply with the observability conditions discussed in the previous sub-section. Considering different initial conditions ($x_0 = [0.03\ 0.02\ 0.01]$ meter) for MM and ($\hat{x}_0 = [0.03\ 0.4\ 0.1]$ meter) for the observer. The states estimation results given by (Fig. 5) are correct, although white noise was added to all the states. Besides, as shown in (Fig. 6), it is clear that estimation error is almost zero. The LMI condition (32) is checked and the obtained matrices P, Q and the observer gain L are:

$$L = \begin{bmatrix} 2.1436 & 0.0023 \\ -0.0216 & 2.0673 \\ -7.1293 & 0.3886\end{bmatrix}, P = \begin{bmatrix} 5.9216 & -0.0667 & 0.3130 \\ -0.0667 & 4.8562 & -0.0195 \\ 0.3130 & -0.0195 & 0.1013\end{bmatrix},$$

$$Q = \begin{bmatrix} 24.5390 & -0.4061 & 4.1753 \\ -0.4061 & 18.0554 & -0.2605 \\ 4.1753 & -0.2605 & 16.6323\end{bmatrix}.$$

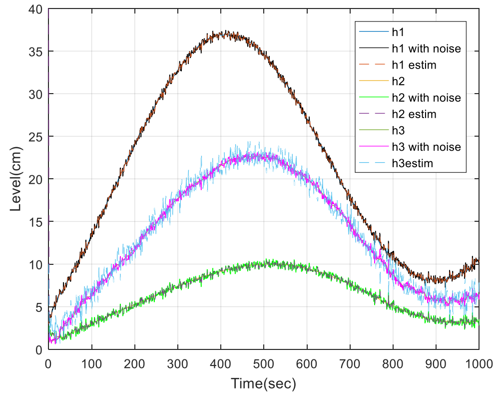


Fig. 5. Comparison of estimated and noisy system states.

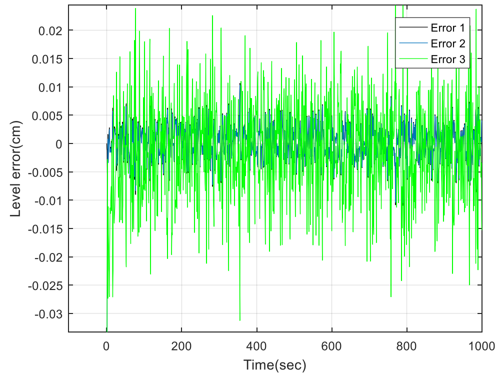


Fig. 6. Estimation errors in the presence of noise.

Fig. 7 shows a comparison between the NL model and the developed MM. For the states h_1 and h_3, it is noted that there is a small error between the NL and the MM states from 0 to 500 sec. This is due to the difference in initial conditions. Fig. 8 shows that all the weighting functions, except for a few peaks due to noise, are in the range $[0\ 1]$, the convexity condition is then respected.

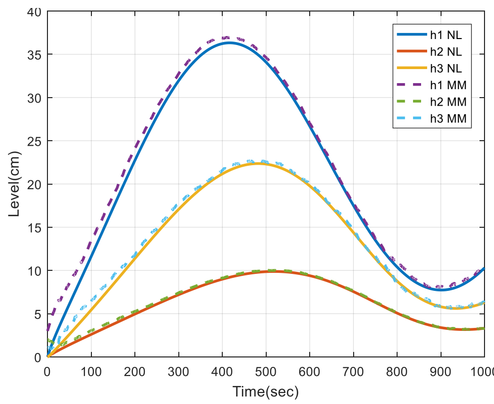


Fig. 7. Comparison between NL model and MM.

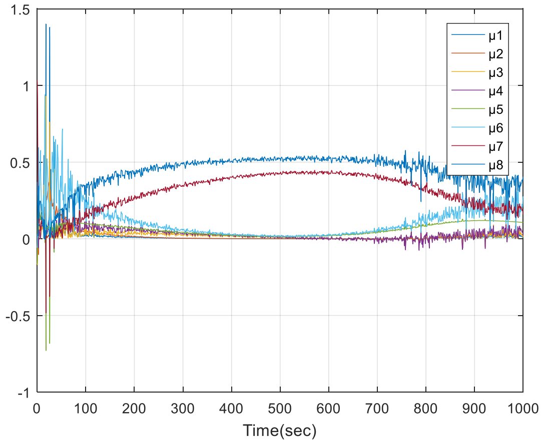


Fig. 8. Weighting functions μ_i

6. MM-qLPV BASED STATE FEEDBACK CONTROL DESIGN

This section is dedicated to the presentation of simulation results obtained by using a MM-qLPV based multivariable state feedback K applied to the control of the non linear three tank system. A standard state feedback control law is given by:

$$u(t) = -Kx(t) \tag{34}$$

In MM convex polytopic approach, one calculates a state feedback gain K_i of each local sub-model. Then, the global gain K is given by the following expressions:

$$K = \sum_{i=1}^{r=8} \mu_i(\hat{x}) * K_i \tag{265}$$

with: $\mu_i(\hat{x})$ the weighting functions given by (23), and the states estimate $\hat{x}$ are ensured by the multi-observer (28). Each matrix gain K_i is calculted using multivariable state feedback algorithm presented in (Antsaklis et al., 1997). The global state feedback controller must ensure the stability of the closed loop system and reduce the effect of exogenous disturbances.

Remark 1 : about the static performance, among the possible solutions to ensure zero static error is the use of decoupling matrix. It is noted that in some applications the use of integrators block is enough to ensure zero static error.

6.1. Closed-loop simulation results (convex polytopic approach)

By using the MM based multi observer (28) and the MM state feedback control (34) and (35), references step change regulation of both levels h_1 and h_2 is performed (in nominal condition). A white noise with zeros mean is added to the outputs. Fig. 9 and Fig. 10 show the simulation results with initial conditions (x_0=*[0.003 0.002 0.001]* meter). The obtained regulation performances are good. However, there is an overshoot in the first output h_1. With the following choice of closed loop poles (*[-0.38 -0.35 -0.36]*), the obtained Ki gains are given by:

$K_1 = \begin{bmatrix} 0.0057 & -0.0017 & -0.0108 \\ -0.0005 & 0.0109 & 0.0567 \end{bmatrix}$; $K_2 = \begin{bmatrix} 0.0060 & 0.0014 & -0.0413 \\ 0.0013 & 0.0114 & -0.1383 \end{bmatrix}$;

$K_3 = \begin{bmatrix} 0.0057 & -0.0017 & -0.0108 \\ -0.0005 & 0.0120 & 0.0567 \end{bmatrix}$; $K_4 = \begin{bmatrix} 0.0060 & 0.0014 & -0.0413 \\ 0.0013 & 0.0125 & -0.1383 \end{bmatrix}$;

$K_5 = \begin{bmatrix} 0.0056 & -0.0007 & -0.0082 \\ -0.0001 & 0.0095 & 0.0676 \end{bmatrix}$; $K_6 = \begin{bmatrix} 0.0056 & 0.0006 & -0.0044 \\ 0.0000 & 0.0103 & -0.0392 \end{bmatrix}$;

$K_7 = \begin{bmatrix} 0.0056 & -0.0007 & -0.0082 \\ -0.0001 & 0.0106 & 0.0676 \end{bmatrix}$; $K_8 = \begin{bmatrix} 0.0056 & 0.0006 & -0.0044 \\ 0.0000 & 0.0114 & -0.0392 \end{bmatrix}$.

Remark 2: it is noted that the static errors are treated by adding the term $u_i = -K_{int} \int_0^t \varepsilon(\theta) d\theta$ to the control $u(t)$, where $\varepsilon(\theta) = y(t) - y_r(t)$, the error between the outputs $y(t)$ and the references $y_r(t)$. After few trials, the following integral matrix gain is chosen: $K_{int} = \begin{bmatrix} 0.0005 & 0 \\ 0 & 0.02 \end{bmatrix}$.

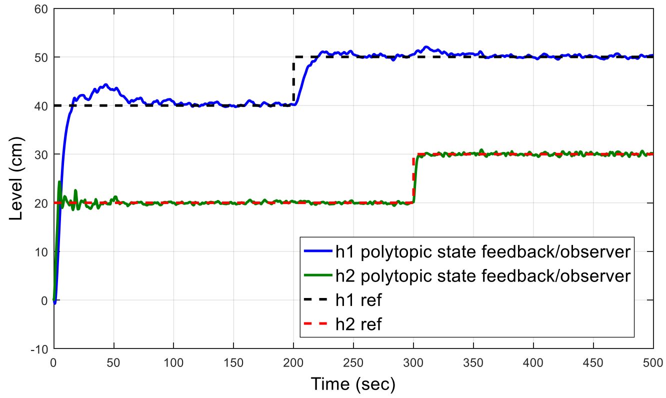
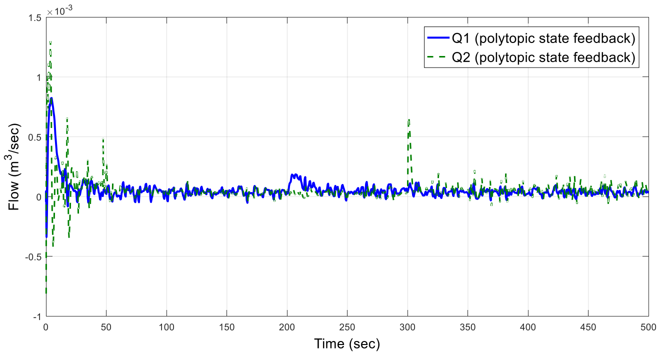
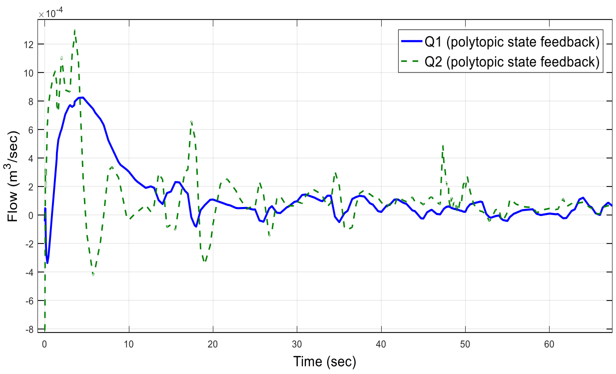


Fig. 9. Three-tank system response in closed loop (polytopic approach with observer and noisy outputs).



(a)



(b)

Fig. 10. (a) State feedback control signals (polytopic approach with noisy outputs), (b) Zoom.

7. MM-qLPV BASED H∞ CONTROL DESIGN

The linear fractional transformations (LFT) of the standard problem is shown in the fig. 11 (Doyle et al., 1991)). The resolution of the standard problem (generalized mixed sensitivity problem) involves finding a dynamic control law $u = K(s)\, y$ minimizing the influence of the exogenous signals w on the controlled output signal $\mathbf{z}$, such that:

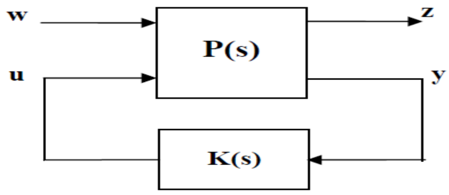


Fig. 11. Standard problem LFT representation.

$$\left\| \begin{bmatrix} W_P S \\ W_a R \\ W_t T \end{bmatrix} \right\|_\infty < 1 \qquad (36)$$

where W_p, W_a, W_t the weighting filters. S, R, T the sensitivity functions. The closed loop states space representation of the three tank system can be written as follows:

$$\begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ \mathbf{z} = C_1 x + D_{11} w + D_{12} u \\ y = C_2 x + D_{21} w + D_{22} u \end{cases} \qquad (37)$$

With : $u = \begin{bmatrix} Q_1 \\ Q_1 \end{bmatrix}, w = \begin{bmatrix} h_1\, ref \\ h_2\, ref \end{bmatrix}, y = \begin{bmatrix} h_1\, mes \\ h_2\, mes \end{bmatrix}$ and $\mathbf{z} = \begin{bmatrix} h_1\, mes \\ h_2\, mes \\ Q_1 \\ Q_2 \end{bmatrix}$

Q_1, Q_2 are the pump flows.

$h_1\, ref$ and $h_2\, ref$ are the reference levels for h_1 and h_2.

$h_1\, mes$ and $h_2\, mes$ are the measurements of levels h_1 and h_2.

The technique adopted in this work to find the robust controller $K(s)$ is based on LMI approach of (Scherer C. et al., 1997). By using the transformation (38), the output feedback controller $K(s)$ is obtained by solving the LMIs (39) and (40). The matrices M and N are chosen such that: $MN^T = I_N - XY$. This choice allows to find the optimal H_∞ controller:

$$\begin{cases} \tilde{D} = D_c \\ \tilde{C} = D_c C_2 X + C_c M^T \\ \tilde{B} = Y B_2 D_c + N B_c \\ \tilde{A} = YAX + YB_2 D_c C_2 X + NB_c C_2 X + YB_2 C_c M^T + NA_c M^T \end{cases} \qquad (38)$$

$$\begin{bmatrix} AX + B_2\tilde{C} & 0 & 0 & 0 \\ \tilde{A} + A^T + C_2^T \tilde{D}^T B_2^T & YA + \tilde{B} C_2 & 0 & 0 \\ B_1^T + D_{21}^T \tilde{D}^T B_2^T & B_1^T Y + D_{21}^T \tilde{B}^T & -\gamma_\infty I_n & 0 \\ C_1 X + D_{12}\tilde{C} & C_1 + D_{12}\tilde{D} C_2 & D_{11} + D_{12}\tilde{D} D_{21} & -\gamma_\infty I_n \end{bmatrix}^H < 0 \qquad (39)$$

$$\begin{bmatrix} X & I_n \\ I_n & Y \end{bmatrix} > 0 \qquad (40)$$

If there exist the matrices $X, Y, \tilde{A}, \tilde{B}, \tilde{C}$ and $\tilde{D}$ with minimal γ_∞. The state representation of the H_∞ controller is given by:

$$\begin{bmatrix} \dot{x}_c \\ u \end{bmatrix} = \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} \begin{bmatrix} x_c \\ y \end{bmatrix} \qquad (27)$$

7.1. Weighting functions selection

Weighting filter W_P :

Used to reject low frequency disturbances,. W_P is chosen as a low pass filter given by:

$$W_P = \frac{K_c s + K_d}{s + K_d} \qquad (42)$$

Where K_c, K_d are the tuning parameters.

Weighting filter W_u:

Used to limit the bandwidth of the controller and avoid high amplitude control signals, W_u is appropriately chosen as a constant gain.

Weighting filter W_t :

Is chosen as a high-pass filter given by:

$$W_t = \frac{K_e s + K_f}{s + K_b} \qquad (43)$$

Where K_e, K_f and K_b are the tuning parameters.

7.2. Simulation results

Remember that the developed MM is made up of eight sub-models, so eight standard representations have been obtained. Now, one calculates for each sub-model an output feedback controller $K_i(s)$ $(i=1,\ldots, 8)$ that ensures both stability, dynamic and static performances (imposed by the weighting filters). The synthesized controller must also be realizable (i.e., the control efforts must not exceed the actuators physical limits). In our case, the maximum effort, which is the pumps maximum flow, is equal to $Qmax=0.01\ m^3/sec$.

Finally, the global H_∞ output feedback controller $K(s)$ is created by a weighting sum combining all the sub-models. Each of the $K_i(s)$ controllers can be written in the form of a dynamic output feedback whose state matrices are given by:

$$K_i(s) = \begin{bmatrix} A_{ci} & B_{ci} \\ D_{ci} & D_{ci} \end{bmatrix} \qquad (44)$$

and the overall polytopic MM-H∞ controller $K(s)$ is given by:

$$K(s) = \sum_{i=1}^{r=8} \mu_i * \begin{bmatrix} A_{ci} & B_{ci} \\ D_{ci} & D_{ci} \end{bmatrix} \qquad (45)$$

With : μ_i the weighting functions satisfying the convexity conditions:

$$0 \le \mu_i(t) \le 1 \quad \text{and} \quad \sum_{i=1}^{r=8} \mu_i = 1 \qquad (46)$$

Finally, to check the performances of the proposed controller/observer synthesis approach, the robust MM-H_∞ controller $K(s)$ is applied to the non-linear model of the three-tank system. The states used in the premise variable ($\mu_i(\hat{x})$) are observed by the developed multi-observer (28). Fig. 12 shows the simulation results of the outputs levels (h_1 and h_2) of the three-tank system non linear model, with initial conditions $x_0 = [0.02\ 0.09\ 0.01]$ meter. Different initial states have been used for the observer $x_0 = [0.03\ 0.02\ 0.01]$ meter. One can see that the output levels follow the references well with a small overshoot for the level h_2 (around 6.05%), no overshoot for h_1. Acceptable response times ($tr_{h1} = 54.7\ sec$, $tr_{h2} = 36.14\ sec$) are obtained. Fig.13 (a, b and c) shows the control signals of the pumps with zooms on the step change instants. These signals are 50% less than the authorized maximum effort.

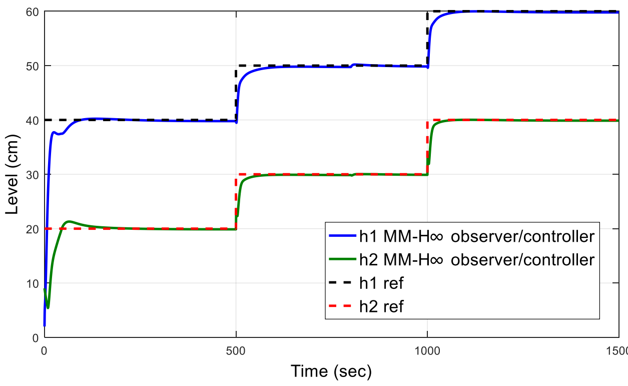


Fig. 12. Three-tank system outputs with MM-H∞ controller/observer.

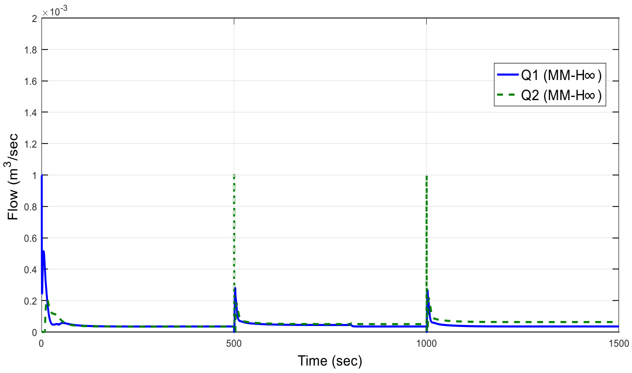


Fig.13. (a) MM-H∞ control signals with observer.

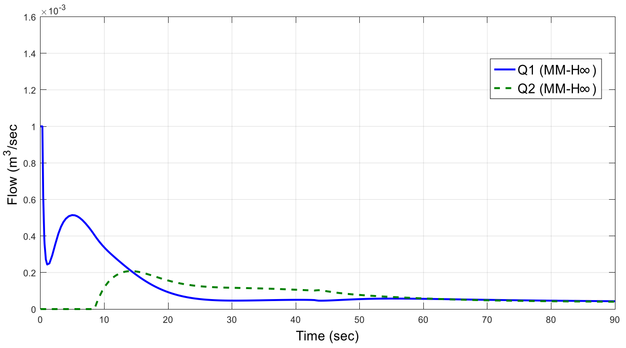


Fig. 13. (b) MM-H∞ control signals (zoom 1)./

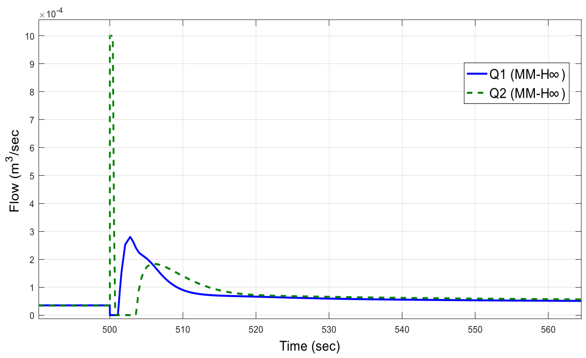


Fig. 13. (c) MM-H∞ control signals (zoom 2).

Fig. 14 shows that the convexity condition (all the weighting functions belong in the interval *[0 1]*) is checked. Fig. 15 shows the same thing for the functions $F_{i,j}$.

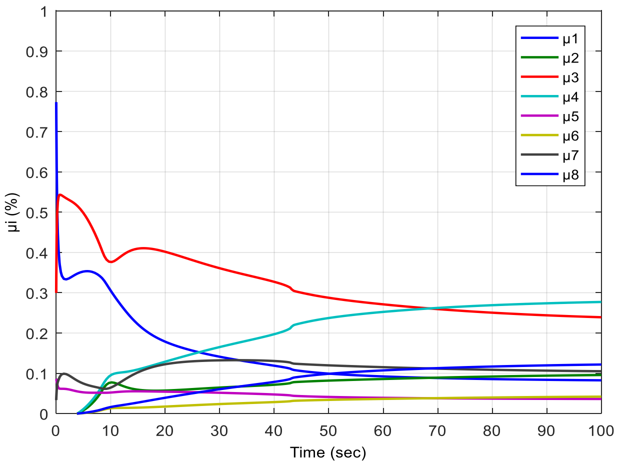


Fig. 14. Weighting functions.

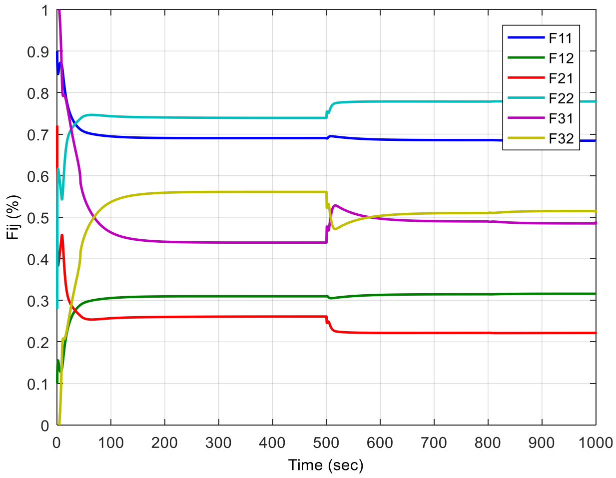


Fig. 15. Functions $F_{i,j}$.

To test the robustness of the proposed design approach in the presence of noise, a white noise (with zero mean) is added to the system outputs. Although the level measurements given by the sensors are noisy, the MM-H_∞ controller/observer ensures a good regulation as shown in fig. 16.

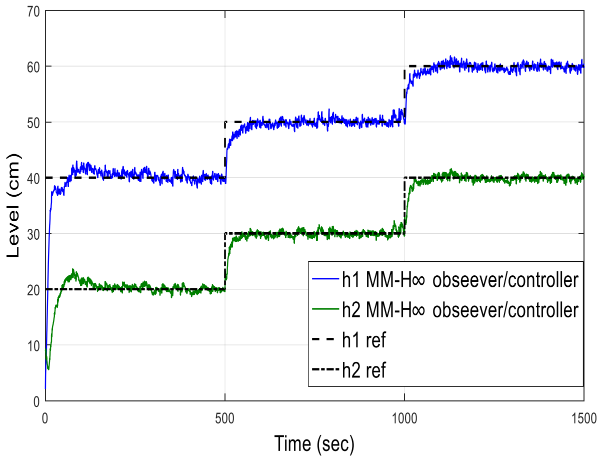


Fig. 16. Three-tank system outputs with MM-H∞ controller/observer (with noise).

8. COMPARISON BETWEEN MULTIVARIABLE STATE FEEDBACK AND MM-H∞ CONTROLLERS

In this section a robustness test is performed to compare the robust performances of both designed controllers (the polytopic state feedback control and the MM-H∞ control) in disturbed mode. Firstly, parametric uncertainty robustness test is carried out, and secondly robustness against external disturbance is tested.

8.1. Parametric uncertainty robustness test

In this case the valve *10* (which was initially closed during all the previous simulation) is *20%* opened in the interval *[500 800]* sec. which causes a change in the corresponding az_{10} parameter. Fig. 17 and fig. 18 show the obtained results.

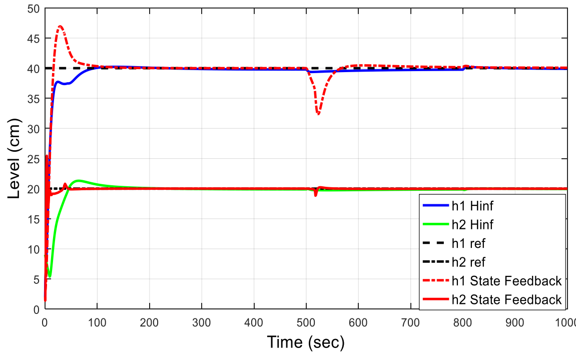


Fig. 17. Comparison between state feedback and MM-H∞.

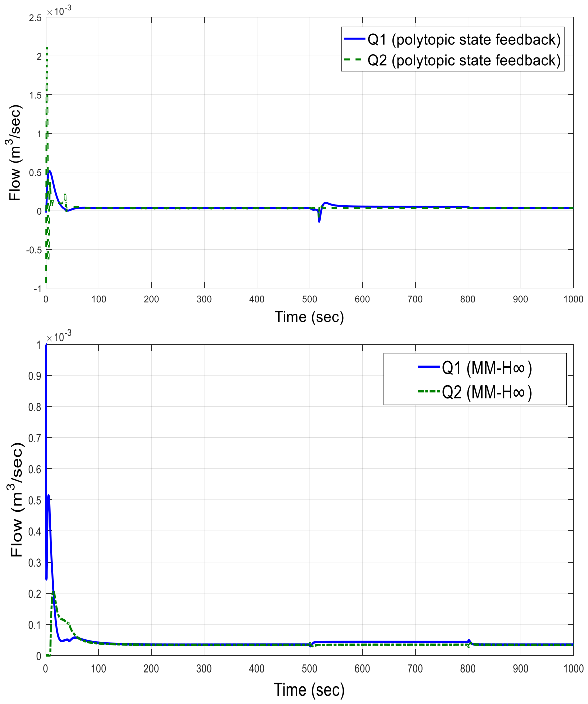


Fig. 18. Polytopic state feedback and MM-H∞ control signals.

One notices that both controllers (polytopic state feedback and MM-H∞) are quite robust against this parameter uncertainty, but the performances of the MM-H∞ controller are better than those of the state feedback. Table 3. summarizes these performances.

Table 3. Performances Comparison between state feedback and MM-H∞ controllers.

performances	State feedback controller	MM-H∞ controller
Overshoot	h_1 = 17.5 % h_2 = 27.5 %	h_1 = 0 % h_2 = 6.05 %
Static Error	0	0
Response time	tr_{h1} = 50.48 s tr_{h2} = 19.15 s	tr_{h1} = 54.7 s tr_{h2} = 36.14 s
Disturbance rejection time	t_{h2} = 8.5 s t_{h1} = 25.1 s	t_{h2} = 4.2 s t_{h1} = 10.6 s

According to this table it is noted that the polytopic state feedback controller has larger overshoot than the MM-H∞ one, but it is faster. The static error is zero in both controllers. Regarding the time of disturbance rejection, it is noted that the robustness of the MM-H∞ controller is better than the polytopic state feedback one.

8.2. Measurement disturbances test

In this case, a disturbance of *2cm* is added to the measurements (h_1 and h_2) from *300 sec* to *500 sec*. Fig. 19 (a and b) and fig. 20 (a and b) show respectively the regulated outputs (h_1, h_2 levels) subject to external disturbances and the corresponding control signals.

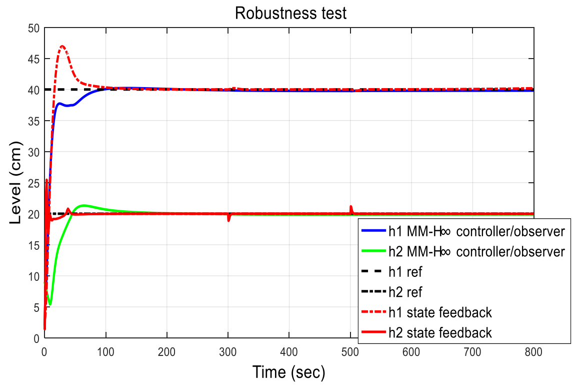


Fig. 19. (a) Comparison between state feedback and MM-H∞ controls subject to external disturbance.

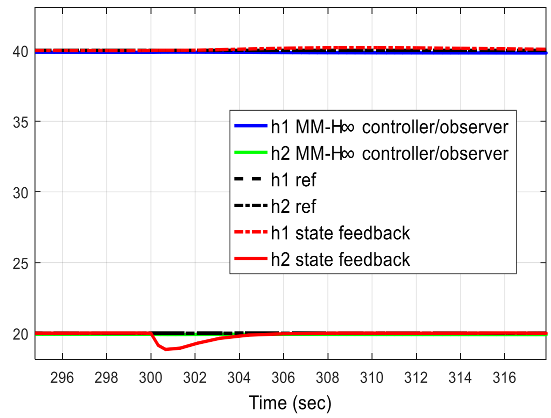


Fig. 19. (b) Comparison between state feedback and MM-H∞ controls subject to external disturbance (Zoom of fig 19.(a)).

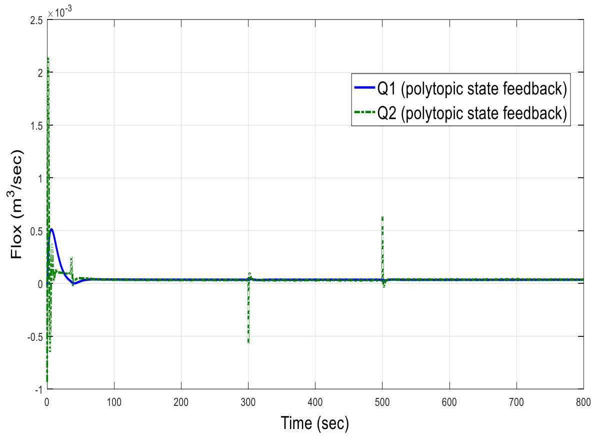


Fig. 20. (a) Polytopic state feedback control signals.

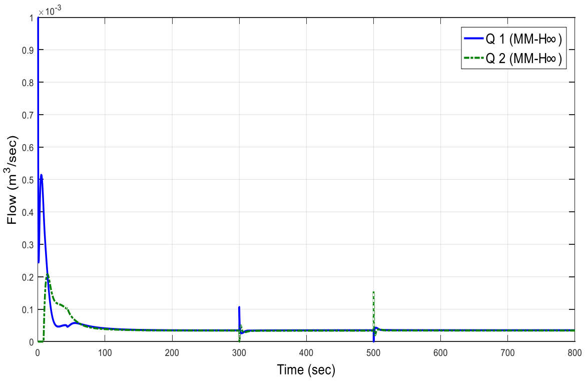


Fig. 20. (b) MM-H∞ control signals.

From these figure, it is noted that the presence of external disturbances influences regulation performances of the polytopic state feedback. On the other hand, the MM-H∞ controller gives better robust performances and keeps some robustness.

9. CONCLUSION

Two control techniques have been developed and applied to the three tanks system: a polytopic state feedback approach and MM-H∞ approach. A comparison of the performances of these techniques is carried out through different simulations test in nominal and disturbed cases. The obtained results have clearly demonstrated the effectiveness of the MM-H∞ controller over the state feedback one in maintaining the robust performances against the system parameters uncertainties, measurement noise and external disturbances of the levels measurements. Finally, it can be concluded, that the robust MM-H∞ controller combined with a multi-observer applied to the three-tank system is efficient, because it can guarantee the achievement of better robust performances compared to state feedback controller. However, it is noted that the tuning of the MM-H∞ waiting filters is obtained after some tests and it could be considered as the main disadvantage of the proposed approach. As a perspective to this work, it will be interesting if the weighting filter parameters could be included as a free parameter in the controller design like (Dinh et al., 2005).

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REFERENCES

AMIRA-DTS2000 (1996). Laboratory setup three tank system. *Amira Gmbh*, Duisburg, Germany.

Antsaklis, P. J. and Michel, A. N. (1997). Linear Systems, *McGraw Hill*, New York.

Bistak P. (2015). Time sub-optimal control of triple integrator applied to real three-tank hydraulic system. *Eurocast 2015. LNCS Springer, Cham*, vol. 9520, 25–32.

Bistak P. (2018). Dynamical feedforward control of three tank system, *Eurocast, Part I, LNCS* 10671, 134–141.

Boyd, S., E. L. Ghaoui, Feron E. and Balakrishnan V. (1994). Linear matrix inequalities in system and control theory. *SIAM Studies in Applied Mathematics*, Philadelphia.

Cao, Y. and Frank, P. M. (2001). Stability analysis of nonlinear time-delay systems via linear T-S fuzzy models. *Fuzzy sets and systems*, 124(2), 213-229.

Chen, J., Qin, L. and Toker, O. (2000). Limitations on maximal tracking accuracy. *IEEE transactions on automatic control*, 45(2), 326-331.

Chi-Tsong Chen (1999), Linear System Theory and Design, *The Oxford Series in Electrical and Computer Engineering*. 3rd Edition. OXFORD University Press.

Chouaba, S. (2012). Contribution à l'estimation des modèles linéaires à paramètres variantes à temps continu. Application à la modélisation des échangeurs de chaleur», Ph. D thesis, *Poitiers university*, France.

Dinh, M., Scorletti, G., Fromion, V. and Magarotto, E. (2005). Parameter dependent H∞ control by finite dimensional LMI optimization : application to trade-off dependent control, *International Journal of Robust and Nonlinear Control*, 15, 383–406.

Doyle, J., Packard, A. and Zhou, K. (1991). Review of LFTs, LMIs and µ. *IEEE Conference on Decision and Control*, Brignton, England, 1227–1232.

Elmira, M. and Yao, D. S. (2017). Model free control approach of a three tank system using an adaptive based control, *Proceeding of the ASME, International Design Engineering Technical Conferences*, Cleveland, Ohio, USA.

Galindo R. (2005). Mixed sensitivity H∞ control of a three-tank-System, *American Control Conference*, Portland, USA.

Hituraj, S. and Ramakalyan, A. (2016). Interval Fuzzy Type II controller for the level control of a three tank system, *I FAC*, 49(1), 561–566.

Ichalal, D., Marx, B., Ragot, J., Maquin, D. (2008). Design of observers for Takagi-Sugeno systems with immeasurable premise variables: an L_2 approach. *In 17th IFAC World Congress*, Seoul, Korea.

Kiss, N., Marx, B., Mourot, G., Schutz G., Ragot J.(2009). State estimation of the three-tank system using a multiple model. *48th IEEE Conference on Decision and*

Control and 28th Chinese Control Conference, Shanghai, 7795-7800.

Mastacan, L. and Dosoftei, C. (2013). Level Fuzzy Control of Three-Tank System, *International Conference on Control Systems and Computer Science (CSCS)*, 30-35.

Mellouli, E., Alfidi, M. and Boumhidi, I. (2018). Fuzzy sliding mode control for three-tank system based on linear matrix inequality, *Int. J. Automation and Control*, 12(2).

Murray, S. R. and Johansen, T. A. (1997). Multiple model approaches to modeling and control. *Taylor & Francis*, London.

Qingfeng, W., Yanli, Y. and Hong C. (2006). H∞ Control of Three-tank System Over Networks with Packet Dropout, *Proceedings of the 6th World Congress on Intelligent Control and Automation*, Dalian, China.

Rodrigues, M. (2005). Diagnostic et commande active tolérante aux défauts appliqués aux systèmes décrits par des multi-modèles linéaires, Ph.D. thesis, *Université Henri Poincaré,* Nancy 1, France.

Sarailooa M. , Rahmanib, Z. and Rezaieb. B. (2015). A model predictive control scheme based on Bees algorithm in a class of nonlinear systems: Application to a three tank system, *Neurocomputing*, 152, 294–304.

Sathishkumar, K. Kirubakaran, V. and Radhakrishnan, T. K. (2017). Real time modeling and control of three tank hybrid system, *Chemical Product and Process Modeling*. Walter de Gruyter GmbH, Berlin/Boston.

Scherer, C. , Gahinet P. and Chilali, M. (1997). Multiobjective output-feedback control via LMI optimization. *IEEE Trans.on Automatic Control*, 42 (7), 896–911.

Scherer, C. W. (2001). LPV control and full block multipliers, *Automatica*, 37, 361-375.

Shuyou, Y., Yu, Z., Yangyang, F., Ting, Q. and Hong, C. (2018). Liquid level tracking control of three-tank systems, *Proc. of the 37th Chinese Control Conference*, China.

Stoustrup, J. and Niemann, H. (2000). Application of an H∞ based FDI and control scheme for the three tank system, *IFAC Fault Detection, Supervision and Safety for Technical Processes*, Budapest, Hungary.

Takagi, T. and Sugeno, M. (1985) . Fuzzy identication of systems and its application to modelling and control. *IEEE Trans. On Systems Man and cybernetics*, 15(1), 116-132.

Zerar M., Cazaurang F. and Zolghadri A. (2009). Coupled LPV and flatness-based approach for space reentry guidance. *Control Theory and Applications*, 3(8), 1081-1092.

Zhenping, Ji and Zhan, S. (2012). The research of predictive control algorithm for the three-tank water level system based on the multi model, *Applied Mechanics and materials*, 220, 1199-1202.

Zhou, F., Peng, H. and Qin, Y.(2015). RBF-ARX model based MPC strategies with application to a water tank system, *Journal of Process Control*, 34, 97-116.

Zi-Jiang, Y. and Hiroyuki, S. (2017). Robust nonlinear control of a three-tank system in the presence of mismatched uncertainties, *IFAC,* 50(1), 4088–4093.

Zi-Jiang, Y. and Hiroyuki, S. (2019). Robust nonlinear control of a three tank system using finite time disturbance observers, *Control Engineering Practice*, 84, 63–71.