MODELING AND SIMULATION OF ELECTRON'S TRAJECTORY INSIDE OF ELECTRON BEAM GUN

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***Abstract**: The paper aims at presenting a modality of simulation for electrons inside the electron beam gun, based on the mathematical model of particles movement in an electromagnetic field. For the initial given conditions, the trajectory of the electrons which cross the magnetic field of focusing coils is determined by solving the movement equations using the Runge-Kutta methods. Thus, it is possible to determine the limit values which allow the control of the electrons focus.*

***Keywords**: electron beam, magnetic field, control system.*

1. INTRODUCTION

Electron Beam (EB) processing has become a vital technology in many industries. In several cases electron beam processing is the only technology that could be used to achieve the instantaneous local melting and vaporization of the workpiece material.

EB processing use a highly focused beam of electrons as a heat source. Usually the electrons are extracted from a hot cathode, accelerated by a high potential, typically (30…200) kV, and magnetically focused into a spot with a power density of the order of 10^8 W/cm^2.

EB equipment typically consist of an electron gun, high voltage power source, vacuum chamber, pumping equipment and control system (Fig.1) [2], [3], [5].

The optimal command systems of the EB processing equipments refer to the control of the focus, deflection and 3D position of the workpiece.

The command of the focus requires the control of the beam movement on *Oz* axis, which is perpendicular on the processing surface *xOy*. The command of the deflection requires the control of the beam movement on *xOy* axes.

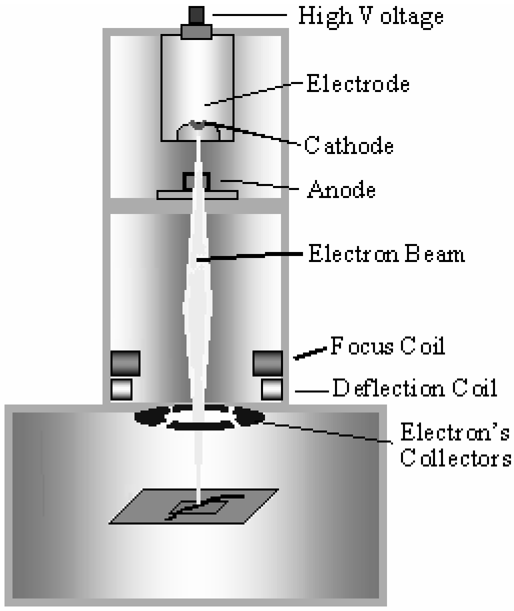


Fig. 1. Electron beam gun

The advance and revolution movements, in vacuum conditions, need to control the system of mechanical positioning for: revolution around of any axis which is perpendicular on the *xOy* axes, with constant angular velocity, translations along the *Ox* and *Oy* axes.

2. MODELING OF THE ELECTRON'S MOVEMENT

The general form of the movement equation for the electrons with *e* charge, considering the resting mass m_0, which are moving with v velocity, into an area where simultaneous exist an electric field of E intensity and an magnetic field of B intensity, is given by the relations:

$$m_0 \frac{d\mathbf{v}}{dt} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{1}$$

$$\frac{d^2\mathbf{r}}{dt^2} = \frac{e}{m_0}(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \tag{2}$$

where **r** is a position vector locating the electron with respect to any origin [1], [10].

For the electrons, the $e/m_0 = 1{,}76\ 10^{11}$ ratio, shows a very great mobility of these. To solve the equation (2) it needs to know the initial conditions of the motion (r_0, v_0). For the description of the motion in the electric and magnetic field it consider only the components of the **v**, **E**, **B** vectors, with direct effect on the electrons. When the electric field is transverse to the magnetic field, the components $\mathbf{E}_y$ and $\mathbf{B}_z$ exist and the equations of motion are:

$$\begin{aligned} \frac{d^2x}{dt^2} &= \frac{e}{m_0}\frac{dy}{dt}\mathbf{B}_z \\ \frac{d^2y}{dt^2} &= \frac{e}{m_0}\mathbf{E}_y - \frac{e}{m_0}\frac{dx}{dt}\mathbf{B}_z . \\ \frac{d^2z}{dt^2} &= 0 \end{aligned} \tag{3}$$

One special solution is obtained for $d^2y/dt^2 = 0$, from which:

$$y_0 = \frac{\mathbf{E}_y}{\frac{e}{m_0}\mathbf{B}_z^2}, \tag{4}$$

The homogeneous solution, respectively the general solution for *y* are (5) and (6):

$$\frac{d^2y}{dt^2} + \left(\frac{e}{m_0}\right)^2 \mathbf{B}_z{}^2 y = 0\,, \tag{5}$$

$$y = \frac{\mathbf{E}_y}{\frac{e}{m_0}\mathbf{B}_z^2} + C_1 \cos\omega_c t + C_2 \sin\omega_c t\,. \tag{6}$$

Similarly, for *x* is obtained:

$$x = \frac{\mathbf{E}_y}{\mathbf{B}_z} t + C_1 \sin\omega_c t - C_2 \cos\omega_c t\,. \tag{7}$$

The values C_1 and C_2 depend on the initial conditions of the electrons to the entrance of the field.

The distribution of the potential inside of the electromagnetic lenses could be approximate through the solution of the Laplace equation:

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \Phi^2} + \frac{\partial^2 V}{\partial z^2}. \tag{8}$$

If it consider only the motion of the electrons in the proximity of the symmetrical axes, the trajectory of these make small angles with the axis, and the potential not depend of the Φ angle, but is function of *r*.

It assume that not exists the motion along Φ, and all the electrons move in *(r,z)* plane (Fig.2).

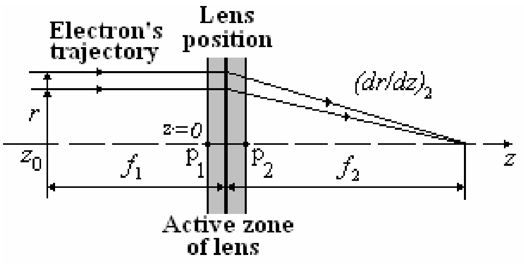


Fig. 2. The trajectory of the electrons which cross the lens

The electrons move in radial plane before and after the lens, but their motions are different because, crossing the lens, the value dϕ/dt modify.

In this conditions, the equations of the movement in cylindrical coordinates, considering the axial symmetry of the magnetic field and the limitation to the lens zone, are:

$$\frac{d^2r}{dt^2} = r\frac{d^2\Phi}{dt^2} - \frac{e}{m_0}\left(\mathbf{B}_\Phi \frac{dz}{dt} - \mathbf{B}_z r\frac{d\Phi}{dt}\right);$$
$$\frac{d^2z}{dt^2} = -\frac{e}{m_0}\left(\mathbf{B}_r r\frac{d\Phi}{dt} - \mathbf{B}_\Phi \frac{dr}{dt}\right). \tag{9}$$

For the magnetic flux **B**, it consider all the components $\mathbf{B}_\Phi$, $\mathbf{B}_r$ and $\mathbf{B}_z$, using the expressions of approximate for the magnetic field. Considering $\mathbf{B}_\Phi = 0$, it determine $\mathbf{B}_r$ in function of $\mathbf{B}_z$ terms, any modification of these leading to correction of $\mathbf{B}_r$. In this conditions, the Laplace equation is:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\mathbf{B}_r) + \frac{\partial \mathbf{B}_z}{\partial z} = 0\,. \tag{10}$$

If the axial component of the magnetic field, $\mathbf{B}_z$, satisfy the Laplace equation, this could be express through the series development (11), with which is obtained the relation (12):

$$\mathbf{B}(z) = \mathbf{B}_0(z) - \frac{d^2}{dt^2}\mathbf{B}_0\frac{r^2}{4} + \ldots \tag{11}$$

$$\mathbf{B}_r = -\frac{d}{dt}\mathbf{B}_0\frac{r}{2} + \frac{d^3}{dt^3}\mathbf{B}_0\frac{r^3}{16} + \ldots \tag{12}$$

Thus, the complete magnetic field could be represented in the terms of axial components and their derivatives, along of the axis. Considering only the first terms of the series development for $\mathbf{B}_z$ şi $\mathbf{B}_r$:

$$\mathbf{B}_z \approx \mathbf{B}_0(z);$$
$$\mathbf{B}_r \approx -\frac{d}{dt}\mathbf{B}_0(z)\frac{r}{2}. \tag{13}$$

it obtains the equation of the motion:

$$\frac{d^2r}{dt^2} = r\left(\frac{d\Phi}{dt}\right)^2 + \frac{e}{m_0}\mathbf{B}_0(z) r\frac{d\Phi}{dt}$$
$$\frac{d^2z}{dt^2} = \frac{e}{m_0}\frac{d}{dt}\mathbf{B}_0(z)\frac{r^2}{2}\frac{d\Phi}{dt} \tag{14}$$
$$\frac{d\Phi}{dt} = -\frac{e}{m_0}\frac{\mathbf{B}_0(z)}{2}$$

respectively:

$$\frac{d^2r}{dt^2} = -\left(\frac{e}{m_0}\right)^2\frac{(\mathbf{B}_0(z))^2}{4}r$$
$$\frac{d^2z}{dt^2} = -\left(\frac{e}{m_0}\right)^2\mathbf{B}_0(z)\frac{d}{dt}\mathbf{B}_0(z)\frac{r^2}{4}. \tag{15}$$

For r^2 of small value, the equations of the motion are:

$$\frac{d^2r}{dt^2} = -\left(\frac{e}{m_0}\right)^2\frac{(\mathbf{B}_0(z))^2}{4}r$$
$$\frac{d^2z}{dt^2} = 0 \tag{16}$$

This approximate allow to determine, in an analytical form, the trajectory of the electrons, if the initial conditions *z(0)* and *r(0)*, and the initial velocities $v_z(0)$ şi $v_r(0)$, are known.

3. SIMULATION OF THE ELECTRON'S MOVEMENT

For the representation of the trajectory of the electrons are determined or known the following dates [4], [11]:

$\mu_0 = 4\pi 10^{-7}$ [Vs/Am] – permitivity of the vacuum;

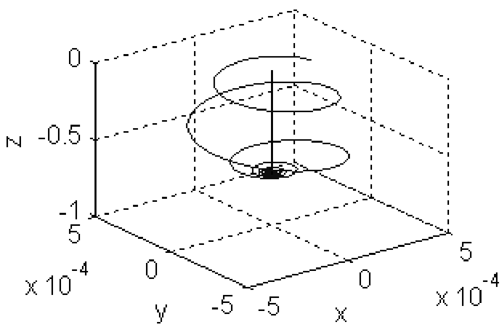
$m_0 = 9{,}108 \cdot 10^{-31}$ [kg] – resting mass of the

electrons;
$e = 1{,}602 \cdot 10^{-19}$ [As] – elementary charge;
$U_{acc} = (40...60)$ [kV] – accelerating voltage;
N=1020 – number of turns of the focus coil;
$k_b = 0{,}6$ – constant value of the magnetic field.

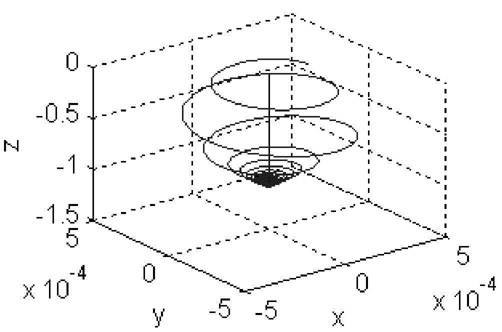
Initial conditions:
z(0) = 0 – origin of the *z* axis;
$v_z(0)$ – velocity which is obtained after the acceleration;
r(0) = 0,001 – position of the electrons near the axis to the entrance in focus coil;
$v_r(0) = 0$ – motion is parallel with *z* axis;
z = 0,5 – position of the focus coil on *z* axis;
t = 0...1e-8 – time domain.

The trajectories of the electrons which cross the magnetic field of the focus coil, are obtained from equation (16), using the Runge-Kutta methods (Fig.3) [12].

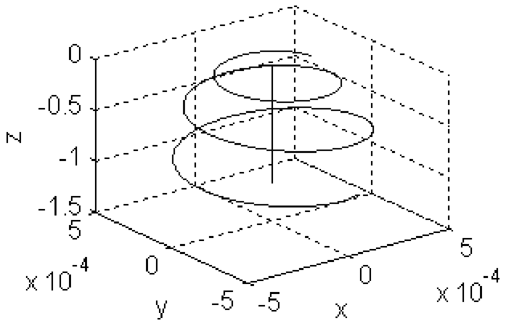
In Figures 3, the origin of the coordinate ϕ is considered to the entrance of the field of the magnetic coil, where the electrons are already on the spiral trajectory.



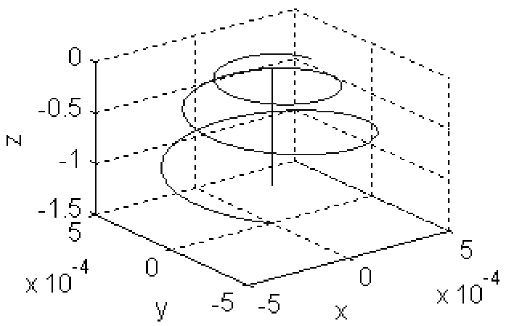
a) U_{acc} = 40 kV, i_{foc} = 0,1 A;



b) U_{acc} = 40 kV, i_{foc} = 0,05 A;



c) U_{acc} = 40 kV, i_{foc} = 0,025 A;



d) U_{acc} = 40 kV, i_{foc} = 0,02 A;

Fig. 3. The trajectory of the electrons which cross the focus coil, at different values for the focus current

The linear velocity is associated with the circular motion, so the total velocity, the circular radius and the focus point fluctuate along the Oz axis.

The position of the focus point with minimum diameter shows the focus distance.

For the considered cases, at the constant value of the accelerating voltage (40 kV), the position of the focus point can be controlled only through the focus current (Fig.3a,b). At the values of the focus current, situated around the 0.025 A and less, the radius of the trajectory grows over the admisible limits and the focus point can't be controlled only through current (Fig.3c,d). In this situations the accelerating voltage allows to control the focus distance but does not allow to control the diameter of the beam. It need to modify the configuration of the focus coil through number of turns and/or geometrical form.

The equations (16) allow, also, to determine the analytical relation for the calculus of the focus distance z_{foc}. Thus, the time is eliminated, known that [1]:

$$\frac{d^2 r}{dt^2} = \frac{d^2 r}{dz^2}\left(\frac{dz}{dt}\right)^2$$
$$\left(\frac{dz}{dt}\right)^2 = -2\frac{e}{m_0}V \tag{17}$$

and results the equation of the trajectory *r(z)*:

$$\frac{d^2 r}{dz^2} = \frac{\frac{e}{m_0} r\mathbf{B}_0^2}{8V}. \tag{18}$$

where V is the electric potential.

By integrating with respect to z, between two points (p_1, p_2, see Fig.2), which limits the active zone of the lens, for the focus distance results:

$$\left(\frac{dr}{dz}\right)_2 - \left(\frac{dr}{dz}\right)_1 = \int_{p_1}^{p_2} \frac{e}{m_0}\frac{\mathbf{B}_0^2 r}{8V} dz. \tag{19}$$

If the lens is sufficiently thin, it could be considered that the electrons doesn't change their distance to z axis. Considering the origin of the z axis ($z_0 = 0$), the focus distance $z_{foc} = f_2$ results:

$$\frac{1}{z_{foc}} = \left.\frac{-\left(dr/dz\right)_2}{r}\right|_{\left(\frac{dr}{dz}\right)_1 = 0} = \frac{-e/m_0}{8V}\int_{p_0}^{p_2} \mathbf{B}_0^2 dz. \tag{20}$$

where the value of the B_0 depends of the focus current i_{foc}, and the geometry of the coil:

$$\mathbf{B}_0(i_{foc}) = \frac{k_b \cdot \mu_0 \cdot i_{foc} \cdot N}{a} \tag{21}$$

with a, the constant of dispersion for the magnetic field, after the symmetrical axis.

The relation for the focus distance, became:

$$z_{foc}\left(i_{foc}, N, a, z_i\right) = \frac{k_b}{-\frac{e}{m_0} \cdot z_i \cdot \left(\mathbf{B}_0(i_{foc}, a, N)\right)^2} \tag{22}$$

In this relation it define the value z_i, called the cross-over distance, measured from the middle plane of the focus coil.

4. CONCLUSIONS

The problem of the electron's movement inside of EB gun is one of analysis and syntesis.

In the design problems, the trajectory of the electrons is given and it is necessary to determine the potentials and the magnetic fields which allow the following of those trajectory, for the given shapes of the electrodes.

When the electrode shapes, potentials and magnetic field configurations are known, it is possible to determine the trajectory of the electrons.

The finding of the solutions for the given trajectory is not unique.

The trajectory of the electrons in magnetic and electric fields, inside of EB gun, is the result of three motions:

- an uniform accelerated motion, along the field **B**, determined by field **E**;
- an uniform motion in the direction normal to the field **B**;
- a circular motion around the axis of the field **B**.

Qualitatively, similar trajectories occur in different geometries in which the fields are crossed.

The total x velocity fluctuates between $\dot{x}_{\min}$ and $\dot{x}_{\max}$, depending on $\mathbf{E}_y / \mathbf{B}_z$ ratio and angular frequency ω_c (see relation 7).

The electronic optics, in case of EB gun is based on the equations of the motion and the concepts borrow from light optics. Thus, it is necessary to exams the electric and magnetic properties of these kind of lenses.

For example, if the value $\mathbf{B_0}$ changes the sign, the equation (18) doesn't change, thus these lenses could be symmetrically.

The verifying of the presented results was possible on the EB gun, from "Petru Maior" University of Tg.Mures (Fig.4).

The EB gun with work chamber, has the following main characteristics:

- triode type CTW 5/60; U_a=60kV;

P_u=5kW; I_{foc} = 85 mA;

- inside dimensions of vacuum chamber (650x650x500) mm [11].

Fig. 4. The EB gun from "Petru Maior" University

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