STABILITY ASPECTS IN DC-DC CONVERTERS USING PID CONTROLLER

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Abstract*-This paper describes a method for stability analysis and design of a fast voltage loop in DC-DC converters. The method is based on the Popov criterion, which unifies tools for nonlinear system analysis with frequency domain analysis tools used in linear systems. This study establishes sufficient conditions for absolute stability of time-varying control systems. The proposed conditions extend in a simple way the classical Popov criterion to time-varying memoryless nonlinearities, using previous results of Kharitonov. They are expressed in terms of Linear Matrix Inequalities (LMIs). The controller design procedure is demonstrated and experimentally verified on a DC-DC converter.*

Keywords: *LMIs, Popov criterion, time-varying systems, buck converter*

1. INTRODUCTION

During the last decade, practical considerations have motivated the study of control systems where the physical parameters are uncertain. One way to represent uncertainties is to let each physical parameter take value independently in an interval. This model, inspired by the paper of Kharitonov (see [1], [10]), has motivated a great amount of research in recent years (see [18], [3], [15]). The so-called interval transfer function family has been extensively studied because of their utility (they easily model more complex uncertainty structure) and because they enjoy powerful extremality properties. For example, in Bhattacharyya et al. [2] robust absolute stability and stabilization criteria for interval transfer function family have been established.

The Popov plot of a transfer function is an invaluable tool in absolute stability theory and adaptive control. Similarly, the collection of Popov plots of the interval transfer function family plays an important role in robustness analysis and synthesis of control systems under nonlinear sector-bounded feedback. In this paper, by exploiting the geometry of Popov plot and related strict positive realness properties, it is shown that, for the collection of Popov plots of interval transfer function family, a large portion of its outer boundary comes from the sixteen Kharitonov transfer functions.

This result is useful in estimating the maximal Popov sector guaranteeing absolute stability of the closed-loop systems

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2. BUCK CONVERTER

In order to dynamically generate a lower supply voltage from a fixed high voltage set by the system, a DC-DC step-down converter is used. The buck converter, shown in Figure 1, is a switching regulator that can efficiently deliver power to a load. Its operation is straight forward and relies on an inductor and capacitor that act as a low-pass filter.

Although there have been significant developments to build inductors and capacitors on-chip [14], [6], current CMOS technology still cannot provide reactive elements that store sufficient energy to efficiently convert power, and therefore this design requires off-chip reactive elements. As long as the switching frequency of the input pulse-width modulated (PWM) rectangular-waves is at least an order of magnitude greater than the cut-off frequency of the low-pass filter, the output voltage of the filter is an average value where its magnitude is set by the duty-cycle of the incoming rectangular-wave.

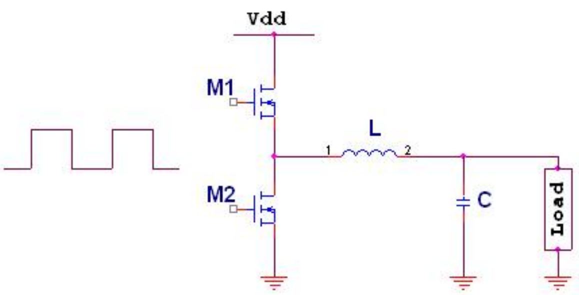

Fig. 1 Buck Converter

Low pass filtering through the inductor and capacitor therefore reduces the AC component of the incoming rectangular wave to an acceptable ripple and its magnitude is set by the ratio of the switching frequency to the filter cut-off frequency. Since the LC filter is a second-order filter, high-frequency AC attenuation is 40 dB/dec. The *pMOS* and *nMOS* transistors are large on-chip devices that chop the input high voltage V_{dd} to generate a rectangular-wave at node V_x with an average voltage that is equal to the desired output voltage with the following equation:

$$V_0 = D \times V_{dd} \tag{1}$$

Therefore, modulating the duty-cycle, D, of the input rectangular wave modulates the regulated output voltage, V_o. These devices also support the average current delivered to the load.

The conversion efficiency of this type of converter approaches 100% as all its components becomes ideal. However, due to several loss mechanisms, efficiency degrades, but values greater than 90 % are still attainable [17], [6]. Using off-chip reactive components can provide very high quality reactive elements with quality factors (Q) greater than 100 and are not the dominant source of loss. Instead, loss is dominated by the resistive losses through the on-chip switching transistors and the power required switching them. The "on" resistance of these devices is inversely proportional to the gate width (*W*).

Load current and switching frequency affect the optimal gate size. Therefore, a designer must consider the power consumption specifications of the load to accurately determine the optimum gate width, which is set by the maximum power requirements. However, for performance driven voltage regulation, power consumption dramatically reduces at lower frequencies due to power's quadratic dependence on voltage. Under lower power conditions, the resistive losses also quickly reduce, but gate switching power remains constant for fixed switching transistor sizes. Appropriately adjusting the widths to be closer to optimal sizing under performance driven load conditions therefore reduces the losses associated with the converter. The optimum gate width varies with load current to achieve higher conversion efficiencies. Switching frequency also affects optimal sizing, but is a nominally fixed parameter constrained by several other factors. Higher switching frequency allows a higher *LC* filter cut-off frequency, which requires smaller inductor and capacitor sizes. This is desirable for portable applications where form factor is a primary concern. However, magnetic saturation of the magnetic core, introduced to increase inductance without affecting the series resistive losses, limits the maximum frequency [17], [14].

Given that the switching frequency is at least an order of magnitude higher than the LC cut-off frequency, a frequency-domain transfer function of the buck converter can be approximated by the following equation:

$$H_{LC}(s)=\frac{\frac{1}{LC}}{s^2+\left(\frac{R_s}{L}+\frac{1}{R_{ld}C}\right)s+\frac{1}{LC}\left(1+\frac{R_s}{R_{ld}}\right)} \quad (2)$$

where the L and C are the inductor and capacitor values, R_S is the series "on" resistance of the switching transistors, and R_{ld} is the resistance of the load chip and dielectric loss of the capacitor at the output. Given the availability of high-quality off-chip inductors and capacitors, there is a resonance at the cut-off frequency due to the complex pole pair of the LC filter. Although high Q's are desirable for efficient power conversion, it can complicate the enclosing control loop design.

Given this mechanism for efficiently delivering power to the load, this adaptive power supply regulation technique needs a way of setting the duty-cycle of the input rectangular wave to regulate the buck converter's output to the desired voltage with respect to some desired frequency of operation.

3. PID CONTROL LOOP

Adaptive setting the duty-cycle of the PWM rectangular to regulate the output voltage with respect to some desired frequency of operation requires a control loop, as shown in Figure 2. It consists of a voltage-controlled oscillator (VCO) that converts the regulated voltage output of the buck converter into a clock signal that oscillates at a voltage-dependent frequency, F_O. This VCO consists of an odd number of inverters in a ring, which oscillates due to positive feedback, and acts to monitor variations in circuit performance relative to process and operating conditions. Taking the difference between the input reference, F_{SP}, and F_O generates an error that feeds into the loop control block. Through negative feedback, the loop locks the output voltage such that the two frequencies match. Therefore, the output voltage tracks with the input frequency reference, where the relationship between the two is dictated by the performance monitoring VCO.

To achieve good transient response characteristics and stability without sacrificing bandwidth, the loop uses proportional, integral, and derivative (PID) control. A frequency-domain model of this PID loop is presented in Figure 2. The resulting open-loop transfer function (loop gain) is as follows:

$$G_{ol}(s)=\left(K_P+\frac{K_I}{s}+K_D s\right)e^{-DTs}H_{LC}(s)K_{VCO} \quad (3)$$

K_P, K_I, and K_D set the pole and zero locations of the proportional, integral, and derivative control block. K_{VCO} represents the oscillator gain *(Hz/V)*. Due to the time required to perform the PID control calculations, its delay *(T)* through the loop causes additional negative phase shift accounted for by the exponential term in the equation.

One difficulty associated with designing this type of controller arises from the resonant peak in the frequency response of the buck converter. For simple integral control, which consists of an integrator followed by the buck converter, there is a potential for instability. An open-loop frequency analysis for this type of loop shows that if the magnitude of the resonant peak crosses above the unity-gain magnitude, negative phase shift due the integrator pole and a pair of poles from the LC filter eliminates phase margin.

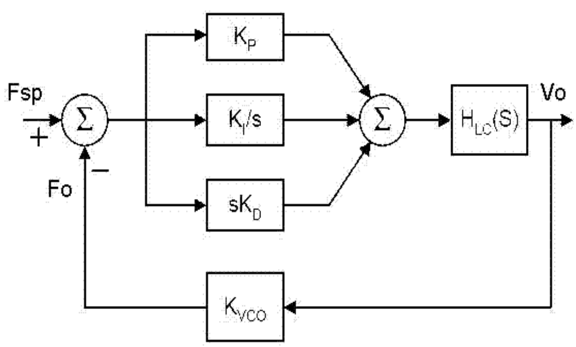

Fig. 2 PID control-loop frequency-domain model

Therefore, the integrator's gain must be sufficiently low as to guarantee that the buck converter's resonant peak never crosses unity gain. Unfortunately, such a configuration leads to low loop bandwidth and slow closed-loop transient response characteristics. To combat this effect, adding a pair of zeros, utilizing proportional and derivative blocks, can stabilize the loop without sacrificing bandwidth. Introducing the zeros at frequencies below the cut-off frequency of the LC filter pushes unity gain crossing of the open-loop response beyond the resonant peak and roles off at *-20 dB/dec*. Furthermore, positive phase shift from the zeros provides sufficient phase margin for a stable loop. The bandwidth of the loop extends beyond what was achievable with integral control alone and the resonant peak of the regulator *LC* is no longer a limiting factor since it occurs below the unity gain frequency. In addition, because the bandwidth exceeds the *LC* filter's cut-off frequency, the loop can quickly respond to sudden load transients that would otherwise perturb the output voltage. This fast response also prevents other noise sources, such as sudden transients in the supply voltage to the buck converter, from propagating to the output.

Implementation of the controller in Figure 2 relies on the ability to generate a PWM rectangular wave, where the duty-cycle *(D)* is the value dictated by the output of a *PID* control block. One possible approach would be to use a frequency detector that compares the incoming reference clock with the output of the oscillator and generate an analog voltage that corresponds to the frequency difference (or error). This error then drives the *PID* control implemented with a set of amplifiers to generate an analog voltage that corresponds to the desired output voltage. Translating this voltage to the appropriate duty-cycle then relies on a comparator that compares a linear ramp wave that has a period equal to the switching frequency of the buck converter to the *PID* control output. While the *PID* output is less than the ramp input, the output of the comparator is low and goes high once the ramp exceeds the PID output. As a result, changing the *PID* output proportionally changes the duty-cycle of the rectangular wave. The enclosing feedback loop compensates for any offsets and non-linearities that may exist in the translation.

4. KHARITONOV'S RESULTS

A polynomial $\alpha(s)$ is said to be stable, denoted by $\alpha(s)\in H$, if all its roots lie within the open left half of the complex plane $(\hat{s}_i \in \Re^-)$. A transfer function $G(s)=\dfrac{\beta(s)}{\alpha(s)}$ is said to be *strictly positive real (SPR),* denoted by $\dfrac{\beta(s)}{\alpha(s)}\in SPR$, if $\alpha(s)\in H$

$$\mathrm{Re}\left(\frac{\beta(s)}{\alpha(s)}\right)>0,(\forall)\omega\in\Re \qquad (4).$$

Consider the *n-th* order interval polynomial family:

$$\Gamma_n=\{\alpha(s)\ |\alpha(s)=\ldots$$
$$\ldots=\sum_{k=0}^{n}a_i s^i, a_i\in\left[a_i^-,a_i^+\right],(\forall)i=\overline{0,n}\} \qquad (5)$$

and denote the four Kharitonov polynomial as:

$$\alpha_{K_1}(s)=a_0^-+a_1^-s+a_2^+s^2+a_3^+s^3+a_4^-s^4+\cdots, \qquad (6)$$

$$\alpha_{K_2}(s)=a_0^++a_1^-s+a_2^-s^2+a_3^+s^3+a_4^+s^4+\cdots, \qquad (7)$$

$$\alpha_{K_3}(s)=a_0^++a_1^+s+a_2^-s^2+a_3^-s^3+a_4^+s^4+\cdots, \qquad (8)$$

$$\alpha_{K_4}(s)=a_0^-+a_1^+s+a_2^+s^2+a_3^-s^3+a_4^-s^4+\cdots. \qquad (9)$$

From Kharitonov's Theorem for the Real Interval Polynomials results following double implication:

$$\begin{gathered}\alpha(s)\in H,(\forall)\alpha(s)\in\Gamma_n\\ \Updownarrow\\ \alpha_{K_1}(s),\alpha_{K_2}(s),\alpha_{K_3}(s),\alpha_{K_4}(s)\in H\end{gathered} \qquad (10)$$

Now consider the strictly proper interval transfer function family:

$$G=\left\{\frac{\beta_{n_v}(s)}{\alpha_{n_u}(s)}\ |\beta_{n_v}\in\Gamma_{n_v},\alpha_{n_u}(s)\in\Gamma_{n_u}\right\} \qquad (11)$$

where $\Gamma_{n_v},\Gamma_{n_v}$ are n -th, n_u-th order interval polynomial families respectively.

Denote their Kharitonov polynomials as $\beta_{K_i^{n_u}}(s),(\forall)i=\overline{1,4}$, $\alpha_{K_j^{n_v}}(s),(\forall)i=\overline{1,4}$ and denote the sixteen Kharitonov transfer functions as:

$$G(s)=\frac{\beta_{K_i^{n_y}}}{\alpha_{K_j^{n_u}}},(\forall)i,j=\overline{1,4} \tag{12}$$

The collection of Popov plots for the interval transfer function family G is denoted as:

$$\begin{aligned}\Pi_G &= \{\mathrm{Re}(G(j\omega))+j\omega\,\mathrm{Im}(G(j\omega)) \mid ...\\ &...\omega\in\Re, G(s)\in G\end{aligned} \tag{13}$$

and the collection of Popov plots for the sixteen Kharitonov transfer functions $G_{ji}(s),(\forall)i,j=\overline{1,4}$ is denoted as:

$$\begin{aligned}\Pi_{G_K} &= \{\mathrm{Re}(G_{ji}(j\omega))+...\\ &..+j\omega\,\mathrm{Im}(G_{ji}(j\omega)),(\forall)i,j=\overline{1,4}\}\end{aligned} \tag{14}$$

Given any convex compact set Π in the complex plane, a point G∈Π is said to be an extreme point of Π, if it cannot be expressed as a convex combination of two distinct points in Π.

The transfer function $G(s)=\frac{\beta(s)}{\alpha(s)}$ is strictly positive real, if and only if

$$\begin{aligned}&\mathrm{Re}\left(\frac{\beta(0)}{\alpha(0)}\right)>0,\\ &\beta(s)\in \mathrm{H},\\ &\alpha(s)+j\eta\beta(s)\in \mathrm{H},(\forall)\eta\in\Re.\end{aligned} \tag{15}$$

For any fixed $k>0$, $\upsilon\in\{-1,1\}$ and $\eta\in\Re$, it follows that:

$$\begin{gathered}\frac{1}{k}+\upsilon(1+\eta s)G(s)\in SPR,(\forall)\psi(s\in\Pi)\\ \Updownarrow\\ \frac{1}{k}+\upsilon(1+\eta s)G_{ji}(s)\in SPR,(\forall)i,j=\overline{1,4}.\end{gathered} \tag{16}$$

It is clear that all the extreme points of Π_G are contained in Π_{GK} (the outer boundary of Π_G). For the collection of Popov plots of the interval transfer function family, a large portion of its outer boundary comes from the sixteen Kharitonov transfer functions and it is useful in estimating the maximal Popov sector with absolute stability. It reduces the verification of Popov criterion for infinitely many transfer functions in the interval family to the verification of Popov criterion for only sixteen critical transfer functions in this family.

5. POPOV STABILITY FOR LTV SYSTEMS

Consider the control system given in figure 3, where G is a strictly proper rational transfer function (matrix) and Φ a time-dependent nonlinearity. This system verifies the following ordinary differential equation:

$$L\left(\frac{d}{dt}\right)y=M\left(\frac{d}{dt}\right)(\Phi(t,y)), \tag{17}$$

where L, M are two coprime real polynomial matrices such that $G(s)=L^{-1}(s)M(s)$. Choosing a minimal representation (A, B, C) of the transfer matrix G, one writes [9]:

$$\dot{x}=Ax+Bu,\quad u=\Phi(t,y),\quad y=Cx, \tag{18}$$

where $x\in\Re^n$, $y\in\Re^{n_y}$, $A\in\Re^{n\times n}$, $B\in\Re^{n\times n_y}$, $C\in\Re^{n_y\times n}$ and $n,n_y\in N^*$.

In the case where Φ is decentralized (that is $(\forall)i,j=\overline{1,n_y},i\neq j\Rightarrow\frac{\partial\Phi_i}{\partial y_j}\equiv 0$, and time-invariant, and verifies [7],

$\Phi(0)=0$ and $(\forall)y,\Phi^T(y)(\Phi(y)-Ky)\le 0$,

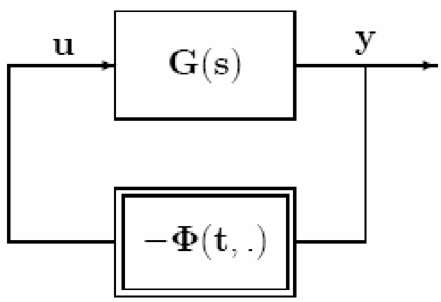

Fig. 3 A nonlinear feedback system

for a certain nonnegative diagonal matrix K, the Popov criterion (see [3], [8], [14]) ensures that the system (17) is asymptotically stable in the large if the roots of L have negative real part and if there exists a constant $\eta\in\Re$ such that

$$I+(I+\eta s)KG(s)\text{ is SPR.} \tag{19}$$

Some attempts have been made to generalize the Popov criterion to time-varying systems. Anderson et al. [1] provide a criterion for systems with nonstationnary linear part and time-independent nonlinearities (the results proposed in the case of time-varying nonlinearities indeed reduce to circle criterion). Narendra et al.[11] are obtained conditions for global stability. They involve two parts: the Popov condition plus a differential (in the case of a separate nonlinearity $\Phi(t,y)=k(t)\times f(t)$) or integro-differential inequality linking $\frac{\partial \Phi}{\partial t}$ and Φ.

Bliman and Krasnosel'skii [3] have obtained an extension of the Popov criterion to nonautonomous systems, more precisely, one provides conditions ensuring local stability of the origin. These conditions are expressed in terms of Linear Matrix Inequalities (LMIs), as a frequency condition, as a graphical condition in the Popov plane. A computational advantage of the LMIs is the fact that they now constitute a standard class of problems, for which recent numerical methods have been developed (see [4], [3], [10], [15]).

In the case when the nonlinearity Φ is decentralized and $\frac{\partial^2 \Phi}{\partial y \partial t}(t,y)\le 0$, and exists a diagonal nonnegative matrix $K \triangleq diag\{k_i\} \in \Re^{n \times n_y}$ the system (17) is asymptotically stable if and only if the following LMIs is feasible:

$$P>0, \eta \ge 0,$$
$$\begin{pmatrix} A^T P + PA & -PB + C^T K + A^T C^T K\eta \\ -B^T P + KC + \eta KCA & -2I - \eta KCB - B^T C^T K\eta \end{pmatrix} < 0 \quad (20)$$

When the zeros of L have negative real part, the frequency interpretation of this condition yields the Popov criterion.

5.1 *Graphical Interpretation of Popov Stability Criterion*

In the SISO case Popov obtained elegant sufficient condition for the global asymptotic stability of system from figure 3. If *G(s)* is a stable transfer function, and Φ is a time-invariant nonlinearity which belong to the *Sector [0, k]*, then a sufficient condition for absolute stability is that there exists a real number η such that

$$\frac{1}{k} + \mathrm{Re}\{(1+\eta j\omega)G(s)\} > 0, (\forall)\omega \in \Re.$$

To illustrate the Popov criterion, it must be generated the Popov plot

$$\tilde{G}(j\omega) = \mathrm{Re}\{G(j\omega)\} + j\omega \times \mathrm{Im}\{G(j\omega)\}.$$

As shown in figure 4, the limiting value of the Popov gain k is obtained by selecting a straight line in the Popov plane such that the Popov plot of $\tilde{G}(j\omega)$ lies below this line.

5.2 *The Robust Absolute Stability Problem*

Now it is extend the classical absolute stability problem by allowing the linear system *G(s)* to lie in a family of systems **G(s)** containing parametric uncertainty. Thus, it is possible to deal with a robustness problem where parametric uncertainty as well as sector bounded nonlinear feedback gains are simultaneously present. For a given class of nonlinearities lying in a prescribed sector the closed loop system will be said to be robustly absolutely stable for every *G(s)* $\in$ **G(s)**. It is presented a constructive procedure to calculate the size of the stability sector using the Popov Criterion when *G(s)* is an interval system or a linear interval system. There is no difficulty to see that an appropriate sector can be determined by replacing the family *G(s)* by the extremal set $G_E(s)$.

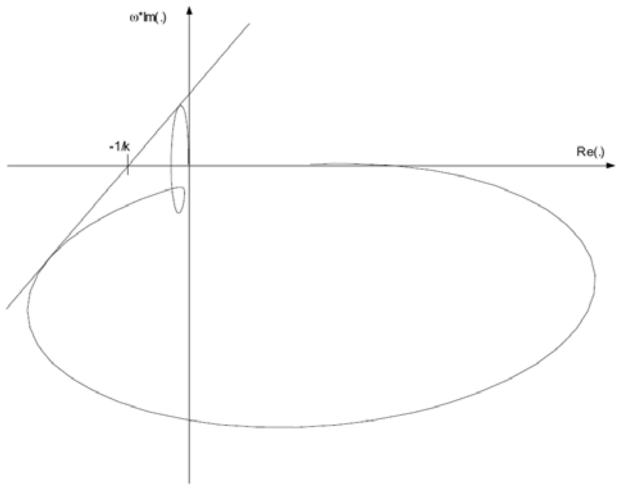

Fig. 4 A graphical interpretation of Popov criterion

It consider the system from figure 3, where the forward loop element *G(s)* lies in an interval family **G(s)** and the feedback loop contains as before a time-varying sector bounded nonlinearity Φ lying in the *Sector [0, k]*. As usual let $G_K(s)$ denote the transfer functions of the

Kharitonov systems associated with the family *G(s)*.

The feedback system in figure 3 is absolutely stable for every *G(s)* in the interval family **G(s)** of stable proper systems, if the time-varying nonlinearity Φ belongs to the *Sector [0, k]* where $k \to \infty, \inf_{G_K} \inf_{\omega \in \Re} \mathrm{Re}\{G(j\omega)\} \geq 0$, otherwise $k < -\frac{1}{ing_{G_K} \inf_{\omega \in \Re} \mathrm{Re}\{G(j\omega)\}}$, where $G_K(s)$ is the set of sixteen Kharitonov systems corresponding to *G(s)*.

6. EXPERIMENTAL RESULTS

The open-loop transfer function (3) is as follows:

$$G(s) = \frac{b_0 s^2 + b_1 s + b_2}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s} e^{-sDT} \tag{20}$$

To obtain the robust Popov gain it must determinate the solution of LMI (20) for the set of sixteen Kharitonov systems corresponding to (21). The result when the Popov criterion is satisfied (stable close loop) is presented in Figure 5.

The result when the Popov criterion is not satisfied (unstable close loop) is presented in Figure 6. In this case the PWM rectangular wave generation is variable.

Fig. 5 PWM rectangular wave for stable closed loop

Fig. 6 PWM rectangular wave for unstable closed loop

7.CONCLUSIONS

In this paper, it was studied a robust stability condition of nonlinear feedback systems and derived its LMI representation in the state space when the linear part of the nonlinear control system is finite-dimension. With this LMI approach, it is possible to get around the difficulty of the convex optimization approach in the frequency domain.
The method for stability analysis of a fast voltage loop controller of buck converter is presented and a set of guidelines for the design and implementation of a fast voltage loop compensator are given. The method, which follows directly from the Popov criterion, is simple to use and is capable of guaranteeing the system stability under all operating conditions.
The proposed stability analysis and voltage loop design guidelines can be used with many different techniques for elimination of the influence of the output voltage ripple on the voltage loop.
The experimental results show fast and stable operation of the voltage loop and verify validity of the proposed method.

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