Nonlinear State Estimation of the Quadruple Tank Process: A Comparison of EKF and ELO Methods

Suguna Arunachalam*, Ranganayaki Velusamy, Deepa Subramaniam Nachimuthu*****

**Government College of Technology, Coimbatore, India*
(e-mail: asuguna@gct.ac.in).
***RVS College of Engineering and Technology, Coimbatore, India*
(e-mail: ranganayaki11@gmail.com)
**** National Institute of Technology Calicut, NIT Kozhikode, India.*
(e-mail: sndeepa@nitc.ac.in)

Abstract: Quadruple Tank Process (QTP) is a multivariable system consisting of four inter connected tanks and two control valves. The quadruple tank system has Minimum Phase (MP) or Non-Minimum Phase (NMP) configurations depending on the two control valve openings. In the QTP system, sensors measure the lower tank levels, while the upper tank levels are not directly measured. State estimation algorithms are employed to monitor the liquid level in four tanks of the QTP. As QTP exhibits multivariable interaction and non-linear dynamics, state estimation is crucial for control. This paper investigates separately the nonlinear state estimation methods for a quadruple tank system under ideal and process uncertainty conditions using Extended Luenberger Observer (ELO) and Extended Kalman Filter (EKF) algorithms. In this paper, the state estimation for a quadruple tank system is carried out for Minimum and Non-Minimum Phase configurations. The ELO and EKF are employed under ideal circumstances and 10%, 20%, and 30% uncertainty are added to the parameters, as changes in the area of drain in tank 1 (a_1) and tank 3 (a_3). Mean Square Error (MSE), Integral Square Error (ISE) and Integral Absolute Error (IAE) are used to evaluate the state estimator's performance in terms of its accuracy in state tracking and its ability to process uncertainty. Simulation results show that under ideal circumstances, EKF and ELO offer good state estimates; however, EKF performs better when there are uncertainties in the process.

Keywords: Quadruple Tank System, Extended Luenberger Observer, Extended Kalman Filter, Process Uncertainty and State Estimation.

1. INTRODUCTION

The system states refer to the variables that, at any given point in time, fully describe the internal condition of the system. State estimators can be static or dynamic, and deterministic or stochastic, providing consistent information about system variables. State estimation, which often relies on mathematical models, provide a reliable representation of the system's internal states, which are frequently not directly measurable. Precise control and monitoring of multi-tank systems are necessary in current industrial applications as they ensure the safety of operations and the high quality of products produced. Among the various types of multi-tank systems, the quadruple tank system has gained prominence as a benchmark for evaluating state estimation techniques due to its complex intrinsic nonlinearity and multivariable dynamics. The QTP system is an excellent illustration of complex industrial processes such as chemical reactors, water treatment plants, and distillation columns, comprised of four interlinked tanks that exhibit complex coupling behaviors. The coupling between the tanks, along with challenges such as non-minimum phase behavior and multivariable interactions, present significant considerations for estimation. To implement advanced control techniques, detect faults and ensure overall process stability, accurate state estimates are crucial in these systems. As a Multi-Input Multi-Output system (MIMO), the quadruple tank system is an essential benchmark for evaluating and managing algorithms because of its nonlinear dynamics. Advances in state estimation technology contribute to improving the overall efficiency, safety, and stability of the system.

Luenberger observer, introduced by (Luenberger, 1971), is designed for state estimation in linear systems. Conversely, the Kalman filter developed by (Kalman, 1960) and later presented by (Welch et al., 1995) is crucial for estimating states in noisy dynamic systems. The filter function has two primary stages: prediction and updation. Initially, it produces a prediction of the system's condition using the most up-to-date estimation, and subsequently, it enhances this forecast using the most recent measurement. The Kalman filter is a tool for exact and continuous state estimation, since it minimizes the error variance in the final state estimate. While the Kalman filter effectively estimates states in a linear system, it struggles with nonlinear dynamics. This restriction is overcome by employing the Extended Kalman Filter, which linearizes the system's equation through Taylor series expansion to handle nonlinearity. Because of its ease of use and reduced computational requirements, the EKF is widely used. In contrast, the Luenberger observer can face challenges when dealing with nonlinear systems. The Extended Luenberger observer strengthens the Luenberger observer by integrating nonlinear observer design and Jacobian matrices to handle nonlinearity. Since most real-time systems are nonlinear, the system states can be estimated using the ELO and EKF.

Many state estimation techniques have been established for the quadruple tank system. (Chandra et al., 2012) introduced an H-infinity filter for estimating liquid levels within a QTP controlled by discrete sliding mode control. Their research emphasized the significance of robust estimating strategies both Non-gaussian and Gaussian noise and demonstrated its effectiveness at a single operating point. However, developing a nonlinear estimator applicable across the entire operating range remains a future research endeavor. (Vijula and Devarajan, 2013) utilized the Kalman filter for accurate liquid level estimation in a multi-variable process characterized by transmission zeros. The combination of a linear quadratic controller and state estimation techniques enabled efficient QTP control, with simulations substantiating its capacity to reject disturbances and track reference signals effectively. Anyway, the Kalman filter was limited to a linear model, making it less capable of capturing the full nonlinear dynamics of the QTP.

(Hedengren et al., 2014) employed an L_1 norm with a dead band to reject noise in the state estimation and predictive control. By adopting this method, the control system's resilience was improved since it avoided the necessity for unnecessary parameter changes. However, its efficacy depends on predefined noise thresholds, which may not perform optimally in environments with frequently changing parameters. (Smida F.et al., 2014) designed an unknown input observer for a quadruple tank system, which allows for simultaneous estimation of unmeasured liquid levels and an unknown flow rate. (Valarmathi, R. et al., 2015) utilized an Unscented Kalman filter (UKF) to find the states of the quadruple tank process system. The research gap is its lack of comparison with other state estimation or hybrid techniques. However, these methods do not discuss the observer's robustness to disturbances and uncertainties in the system dynamics.

(Azam, 2017) investigated the application of static and dynamic Kalman filters in a modified QTP. The study involved introducing disturbances in the form of step changes during simulations. They linearized and discretized the nonlinear state-space model as part of their work. They also showed how Kalman filters may effectively handle disturbances in modified QTP systems. However, their dependency on linear models limited their robustness in practical nonlinear scenarios. The state estimation method for stochastic and deterministic discrete-time linear systems is developed by (Aljuwaiser et al., 2018). This state estimator was designed with pole placement strategies, addressed inequality constraints, offered a resilient alternative to conventional methods. While it demonstrated potential for complex control systems, it remained limited to linear dynamics, making it less effective for nonlinear scenarios.

Robust state estimation was achieved by employing techniques such as Taylor series expansion, computation of the Jacobian matrix, and utilizing an H-infinity Kalman filter in (Rigatos et al., 2018). Its effectiveness is limited when faced with significant modeling uncertainties. (Meng et al., 2020) studied disturbance rejection in control systems by developing a disturbance observer that estimates uncertain disturbances using L2-gain disturbance attenuation technology. This method proved particularly effective when disturbances could significantly impact system performance. However, its capacity to adapt to time-varying or unmodelled nonlinearities was limited.

(Zhou et al., 2020) have proposed a technique to estimate the range of possible values for state and sensor flaws in a QTP system. This method is accomplished by employing unknown input decoupling approaches that rely on interval hull calculation methodology. Although effective for handling uncertainties, the approach required precise interval bounds, which may be challenging to determine in real-world systems. (Gurjar et al., 2021) developed an estimator to determine the control valve ratio and employed a sliding mode observer for the estimation of the unmonitored state. They also explored higher-order sliding mode control for the QTP. However, tuning sliding mode parameters remained challenging for the complex nonlinear system.

(Sim, X. W et al., 2023) focused on the state estimation of the four-tank system to estimate liquid levels by minimizing the squared error. This process is carried out using a stochastic optimal control framework, where pump voltages serve as the control inputs. (Wafi and Widjiantoro, 2023) integrated a Luenberger estimator with decentralized control for the QTP, utilizing sensor networks for state estimation. However, they faced challenges due to inter-network communication delays and the effect of nonlinearities.

This comprehensive literature review focuses on the notable progress made in state estimation techniques, specifically about the QTP. The literature review reveals that numerous state estimation techniques have been employed in various quadruple tank system schematics, primarily focus on regulating the Tank 1 and Tank 2 liquid level. Most studies rely on the linear model of the quadruple tank process system, often exploring linear state estimators, such as the Kalman filter and H-infinity filter, typically for either minimum phase or non-minimum phase configuration. While certain literature addresses state estimation with Gaussian noise, a notable gap exists concerning nonlinear state estimators and process uncertainty in the quadruple tank process.

This work addresses the existing gap by developing nonlinear state estimators for the QTP under ideal and process uncertainty conditions. Due to the MIMO nature of the system and the cross-coupling between the tanks, the quadruple tank system nonlinear model is adopted for state estimation techniques. Two non-linear state estimation techniques, the ELO and the EKF are proposed and implemented for Minimum Phase and Non-Minimum Phase configurations of QTP.

The chosen model for this research work is Johanson's quadruple tank system, which provides a solid foundation for evaluating the proposed non-linear estimators under realistic conditions. This contribution not only advances state estimation strategies for QTP but also broadens their applicability by addressing both noise and process uncertainties, a critical step forward in robust control, fault detection and fault tolerant control system.

A key novelty of this work lies in the systematic incorporation of process uncertainty into the state estimation framework. Specifically, the ELO and EKF algorithms are formulated to estimate the states of the nonlinear QTP under process uncertainty. Unlike traditional approaches that predominantly address state estimation under ideal conditions, this research investigates the state estimator performance in the presence of various process uncertainties. These uncertainties are introduced by altering the outlet hole area by ±10%, ±20% and ±30% from their nominal values, reflecting realistic deviations in physical parameters due to operational degradation. To the best of our knowledge, this study is among the first to rigorously analyze nonlinear state estimation for the QTP while explicitly considering such parameter-driven uncertainties. Furthermore, the performance analysis includes a detailed assessment of MSE, ISE and IAE across different levels of uncertainty for various system configurations.

The structure of the paper is as follows: The mathematical modeling of QTP is presented in section 2. Extended Luenberger Observer and Extended Kalman Filter algorithm for state estimation is described in section 3. A detailed result analysis is made by comparing the performance of ELO and EKF using MSE, ISE and IAE is provided in Section 4. The conclusion of the paper is given in section 5.

2. MATHEMATICAL MODELING

(Johansson, 2000) established a complex non-linear mathematical model for the quadruple tank process system, which serves as a standard system for validating state estimation algorithms in MIMO systems. The quadruple tank process system is illustrated in Figure 1 and its mathematical model in (1)-(4). The QTP has four coupled liquid tanks, two control valves, and two pumps. The output of this system are the liquid levels in the two bottom tanks, which are affected by the inputs specifically, the voltages delivered to the pumps. Tanks 1 and 2 are situated beneath Tanks 3 and 4, respectively, and gravity causes the liquid to flow from the top tanks to the corresponding lower tanks. The liquid discharged from the lower tanks flows continually into a reservoir situated below them.

The pumps are crucial in this system as they move liquid from the reservoir back into the tanks. Pump 1 uses a three-way Control Valve 1 (CV_1) to allocate the liquid between Tanks 1 and 4, while Pump 2 utilizes Control Valve 2 (CV_2) to transfer the liquid to Tanks 2 and 3. The valve's placement governs the liquid distribution between the tanks, dictating the flow ratios. The flow ratios for CV_1 and CV_2 are denoted by γ_1 and γ_2, respectively.

The flow ratios significantly influence the system's dynamic characteristics, defining whether the system runs in an MP or NMP configuration. The flow ratio is split such that γ_1 and $(1-\gamma_1)$ are directed to Tanks 1 and 4 respectively, while γ_2 and $(1-\gamma_2)$ are directed to Tanks 2 and 3, respectively. If $0 < \gamma_1 + \gamma_2 < 1$, then the quadruple tank system is in NMP, resulting in a situation, where the upper tanks get more liquid than the lower tanks. If $1 < \gamma_1 + \gamma_2 < 2$, then the quadruple tank system is in MP, indicating a more significant liquid inflow into the lower tanks than the upper tanks.

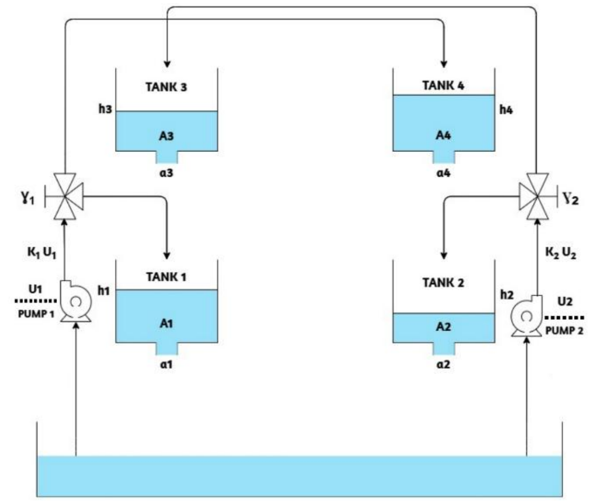

Fig. 1. Quadruple Tank Process.

Based on the principle of mass balance and Bernoulli's law, the mathematical modeling of the Quadruple Tank Process is developed using a traditional fluid dynamics approach. According to the mass balance principle, the difference between the inflow and outflow of each tank, the tanks accumulation rates are given by the equalities.

$$\frac{dh_1}{dt} = \frac{\gamma_1 K_1 u_1}{A_1} + \frac{a_3}{A_1}\sqrt{2gh_3} - \frac{a_1}{A_1}\sqrt{2gh_1} \tag{1}$$

$$\frac{dh_2}{dt} = \frac{\gamma_2 K_2 u_2}{A_2} + \frac{a_4}{A_2}\sqrt{2gh_4} - \frac{a_2}{A_2}\sqrt{2gh_2} \tag{2}$$

$$\frac{dh_3}{dt} = \frac{(1-\gamma_2) K_2 u_2}{A_3} - \frac{a_3}{A_3}\sqrt{2gh_3} \tag{3}$$

$$\frac{dh_4}{dt} = \frac{(1-\gamma_1) K_1 u_1}{A_4} - \frac{a_4}{A_4}\sqrt{2gh_4} \tag{4}$$

where A_i , a_i and h_i ($i = 1..4$) are the tank area, the outlet hole area, and the level of the tank respectively. K_j , u_j, γ_j ($j = 1,2$) and g are the pump proportionality constant, pump input voltage, flow ratio and gravitational constant respectively. The steady-state parameters of the QTP are provided in the Table 1.

Table 1. Steady state parameters of QTP system.

Variables	Description	Values
A_1, A_3	Tank 1 area, Tank 3 area	28 [cm^2]
A_2, A_4	Tank 2 area, Tank 4 area	32 [cm^2]
a_1, a_3	Outlet hole area 1, Outlet hole area 3	0.071 [cm^2]
a_2, a_4	Outlet hole area 2, Outlet hole area 4	0.057 [cm^2]
g	Gravitational constant	981[cm/s^2]
(h_1^0, h_2^0)	Steady state level in Tank 1 and Tank 2	MP: (12.4,12.7) [cm] NMP: (12.6,13.0) [cm]
(h_3^0, h_4^0)	Steady state level in Tank 3 and Tank 4	MP: (1.8,1.4) [cm] NMP: (4.8,4.9) [cm]

K_1, K_2	Pump proportionality constant	MP: (3.33,3.35) [cm³/Vs] NMP:(3.14,3.29) [cm³/Vs]
(u_1^0, u_2^0)	Pump input voltage	MP:(3.00,3.00) [V] NMP: (3.15,3.15) [V]
K_p	sensor gain (p=1,2)	0.50 [V/cm]
γ_1, γ_2	Flow ratio	MP: (0.70,0.60) NMP: (0.43,0.34)

The QTP model (1)-(4) is a non-linear continuous-time system described by a set of first-order differential equations. The system (5) is obtained by discretizing it using the first order Euler's approximation with a constant sampling time T = 0.1 seconds.

$$x_{k+1} = x_k + T\begin{bmatrix} \frac{\gamma_1 K_1 u_{1k}}{A_1} + \frac{a_3}{A_1}\sqrt{2gx_{3k}} - \frac{a_1}{A_1}\sqrt{2gx_{1k}}) \\ \frac{\gamma_2 K_2 u_{2k}}{A_2} + \frac{a_4}{A_2}\sqrt{2gx_{4k}} - \frac{a_2}{A_2}\sqrt{2gx_{2k}}) \\ \frac{(1-\gamma_2)K_2 u_{2k}}{A_3} - \frac{a_3}{A_3}\sqrt{2gx_{3k}}) \\ \frac{(1-\gamma_1)K_1 u_{1k}}{A_4} - \frac{a_4}{A_4}\sqrt{2gx_{4k}}) \end{bmatrix} = f(x_k, u_k) \tag{5}$$

Above, k is the current discrete time, $x_k = [\,h_{1k}, h_{2k}, h_{3k}, h_{4k}\,]^T$ and $u_k = [\,u_{1k}, u_{2k}]^T$ represent the state and input of the discrete system. It is further assumed that the first two components of x_k are measured in order to implement ELO and EKF. The operation is described by Equation (6) where $g(x_k)$ is the measurement function and v_k is a zero mean Gaussian white measurement noise.

$$z_k = g(x_k) + v_k \tag{6}$$

$$g(x_k) = [\,x_{1k}, x_{2k}]^T$$

$$v_k = [\,v_{1k}, v_{2k}]^T$$

3. STATE ESTIMATION ALGORITHMS

3.1 Extended Luenberger Observer

According to the previous section, QTP can be associated, in the absence of measurement noise, with the discrete-time model (7):

$$x_{k+1} = f(x_k, u_k) \tag{7}$$

$$z_k = g(x_k)$$

In general, the control strategies of QTP typically rely on state variable feedback. Since (7) provides feedback only after x_1 and x_2 , it is necessary to provide estimation structures of state variables x_3 and x_4 too. In this context, full order state estimator will be utilized.

It is common practice to employ the Luenberger observer to estimate the states of a linear system. (Alhajeri et al., 2021) presented an ELO, which is an extension of the Luenberger Observer based on the process's linear approximation. Figure 2 depicts the block diagram of discrete-time ELO. The ELO technique aims to estimate the states of a nonlinear system by using output measurement and the system model as the basis for the estimation. The state and output equations of the ELO are presented in (8) with $\hat{x}_k$ and $\hat{z}_k$ denoting the estimates of x_k and z_k and K is the Observer gain.

$$\hat{x}_{k+1} = f(\hat{x}_k, u_k) + K(z_k - \hat{z}_k\,) \tag{8}$$

$$\hat{z}_k = g(\hat{x}_k)$$

Defining the estimation error as in (9), the state equation of the estimator takes the form (10).

$$e_k = x_k - \hat{x}_k \tag{9}$$

$$e_{k+1} = f(x_k, u_k) - f(\hat{x}_k, u_k) - K(z_k - \hat{z}_k\,) \tag{10}$$

ELO gain computation:

The nonlinear quadruple tank process model (1) - (4) is linearized around the steady state operating point in Table 1. This results in the continuous time state space model with matrices A and C as shown in (11), (12).

$$A = \begin{bmatrix} \frac{-1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & \frac{-1}{T_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & \frac{-1}{T_3} & 0 \\ 0 & 0 & 0 & \frac{-1}{T_4} \end{bmatrix} \tag{11}$$

here the time constants are

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}} \qquad i = 1 \dots 4$$

The tank 1 and tank 2 liquid levels are measured using the sensors, then output matrix is

$$C = H = \begin{bmatrix} K_p & 0 & 0 & 0 \\ 0 & K_p & 0 & 0 \end{bmatrix} \tag{12}$$

K_p is the sensor gain (p=1,2). System matrix (A) discretized using the sampling time T as follows:

$$F_d = e^{AT}$$

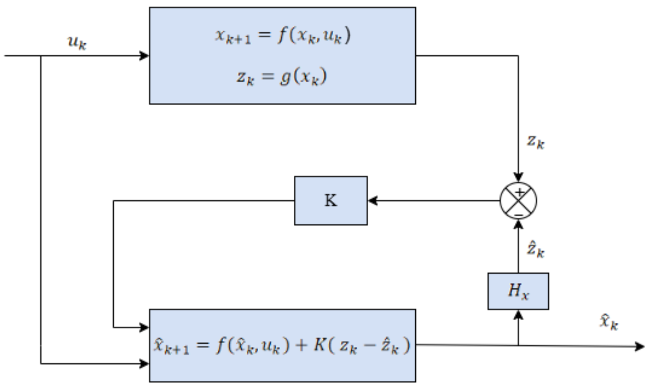

Fig. 2. Extended Luenberger Observer - Block Diagram

The ELO gain K is computed (13) and dynamic error is determined in (14).

$$K = place\,(F_d{}^T, H^T, desired\ poles)^T \tag{13}$$

$$e_{k+1} = (F_d - K\,H)e_k \tag{14}$$

To ensure the stability, K is designed such that the matrix $(F_d - KH)$ has the real positive eigenvalues lie within the unit circle, ensuring error convergence. In the ELO $\hat{x}_k$ is updated using a constant gain matrix K.

3.2 Extended Kalman Filter

The Kalman filter is a widely used algorithm for state estimation in linear systems. Most processes that occur in real life are nonlinear system, such as QTP, which limits the applicability of Kalman Filter. To overcome this, the EKF described by (Simon, 2006), extends the Kalman filters to nonlinear systems. EKF is an extension of the standard Kalman filter, where the nonlinear QTP model is linearized at each time step using Jacobian matrices to estimates system states. The block diagram of discrete time EKF is shown in Figure 3. (Chandra and Gu, 2019) explain that EKF has two stages prediction and updation. The initial state vector $\hat{x}_{0|0}$ is assumed to be zero and the initial covariance matrix $P_{0|0}$ is set to I_4.

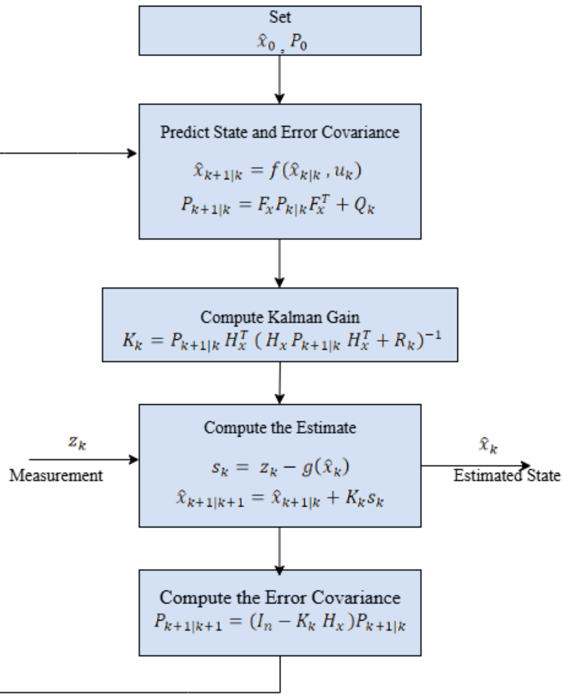

Fig. 3. Block Diagram of the EKF.

Prediction Stage:

State and covariance matrix are as follows in (15) and (16)

$$\hat{x}_{k+1|k} = f(\hat{x}_{k|k}, u_k) \tag{15}$$

$$P_{k+1|k} = F_x P_{k|k} F_x^T + Q_k \tag{16}$$

where, $\hat{x}_{k+1|k}$ is the state vector at time $k+1$ given the state at time k. $P_{k|k}$ and $P_{k+1|k}$ are the previous and predicted error covariance matrix. Q_k is the process noise covariance matrix. F_x denote the Jacobian matrix of $f(x_k, u_k)$ with respect to the x.

$$F_x = \frac{\partial f(x_k, u_k)}{\partial x_k}$$

Updation Stage**:**

The Extended Kalman filter gain matrix K_k (17) is computed

$$K_k = P_{k+1|k} H_x^T (H_x P_{k+1|k} H_x^T + R_k)^{-1} \tag{17}$$

R_k is the measurement noise covariance matrix. H_x denote the Jacobian matrix of $g(x_k)$ with respect to the x.

$$H_x = \frac{\partial g(x_k)}{\partial x_k} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

The updated state and the covariance matrix are obtained using (18) and (19)

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_k s_k \tag{18}$$

where the innovation (residual) vector s_k

$$s_k = z_k - g(\hat{x}_k)$$

$$g(\hat{x}_k) = [\hat{x}_{1k}, \hat{x}_{2k}]^T$$

$$P_{k+1|k+1} = (I_n - K_k H_x) P_{k+1|k} \tag{19}$$

$\hat{x}_{k+1|k+1}$ and $P_{k+1|k+1}$ denote the updated state estimate and covariance matrix respectively. I_n represents the identity matrix with n=4 for QTP system. The Extended Kalman gain is computed based on the error covariance and the state estimate. This recursive process continues at each time step, where the state and covariance matrix are updated based on new measurements. The Kalman gain is updated iteratively so that the state estimation error of the EKF converges towards zero as described by the error dynamics in (10).

The difference between ELO and EKF lies in how the gain matrix is calculated, which weighs the innovation vector $z_k - \hat{z}_k$ components.

4. SIMULATION RESULTS AND DISCUSSION

4.1 Simulation Results

The block diagram of nonlinear state estimator design for QTP is illustrated in Figure 4. The nonlinear state estimation methods, ELO and EKF are implemented separately for QTP under both process uncertainty and normal conditions when the system operates on the minimum and non-minimum phase configurations.

In the quadruple tank system, the liquid level in the lower two tanks (Tanks 1 and 2) are measured using liquid level sensors, while the liquid level in the tanks 3 and 4 is estimated using the ELO and EKF methods. To mimic real-time operating conditions, noise is added to the sensor measurements of tank level. In addition to measurement noise, process uncertainty is also taken into account to evaluate the performance of the nonlinear state estimation method. These uncertainties are modeled as variations of 10%, 20%, and 30% in the outlet cross-sectional area of tank 1 (a_1) and tank 3 (a_3).

When process uncertainty is introduced into the outlet hole areas a_1 and a_3, it affects the liquid level in tanks 1 and 3. By estimating all four states, both the ELO and EKF methods can reflect these changes caused by uncertainties, thereby enabling comprehensive monitoring and visualization of the system.

The simulations have been conducted for equilibrium response, normal and process uncertainty conditions of QTP. A critical performance evaluation of the state estimator is evaluated using the metrics such as MSE, ISE and IAE.

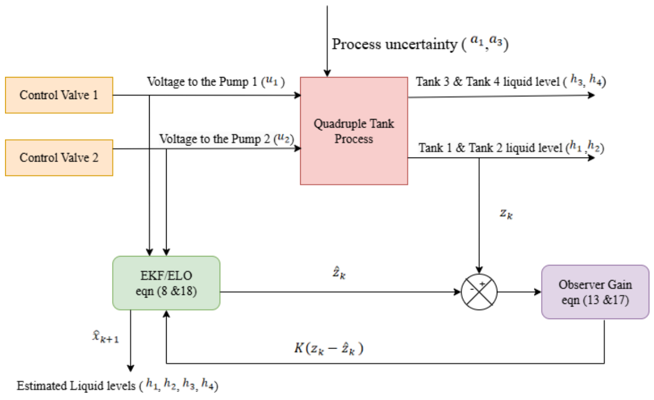

Fig. 4. Nonlinear state estimator design for QTP – Block diagram.

Scenario 1: Equilibrium response of QTP

The objective of this study is to demonstrate the transient and steady state response of the nonlinear QTP under MP and NMP configuration. The initial state vector and input voltage values are defined analytically as follows:

$$x_k = [\,0, 0, 0, 0\,]^T$$

$$u_k = [\,u_{1k}, u_{2k}]^T = \begin{cases} [\,3.00, 3.00]^T\ [V] \text{for MP} \\ [\,3.15, 3.15]^T [V] \text{ for NMP} \end{cases}$$

Other steady state parameters are referenced from Table 1. The simulation results presented in Figures 5 and 6 illustrate the initial state of the NMP and MP configurations of the QTP, where the system is in equilibrium before any changes in control inputs or disturbances are applied. The plotted results show the transient phase, during which the liquid level in tanks increases from zero, followed by the equilibrium response, where the liquid level stabilizes.

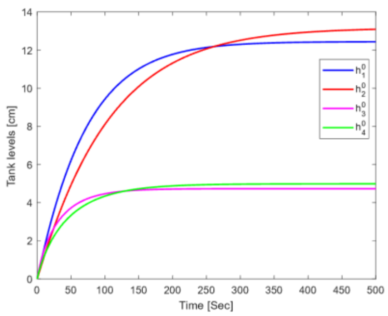

Fig. 5. Transient and steady state response of tank levels in non-minimum configuration of the QTP.

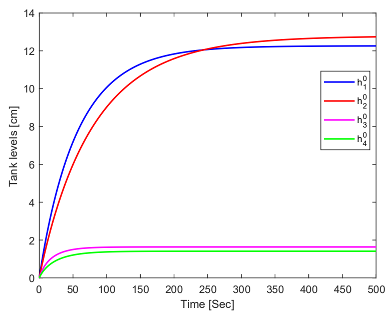

Fig. 6. Transient and steady state response of tank levels in minimum configuration of the QTP.

Scenario 2: Normal Conditions

The objective is to estimate the states of the QTP under normal conditions for both Non- Minimum Phase and Minimum Phase configurations. Figure 7 illustrates the input voltage applied to the pumps for MP and NMP configurations. The input values are varied in a stepwise manner to create realistic testing conditions for the ELO and EKF based state estimation.

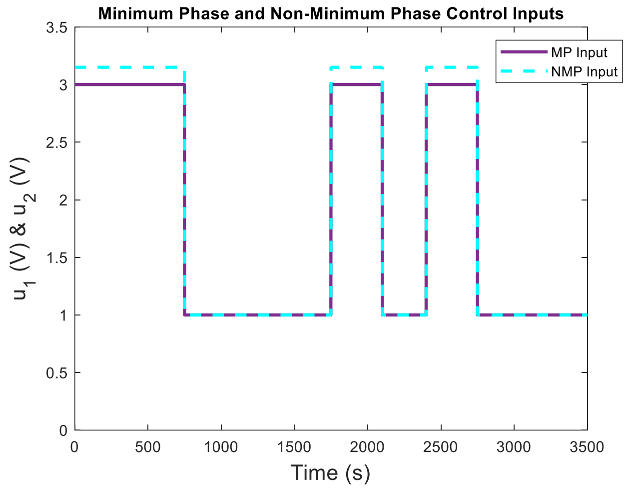

Fig. 7. Input - Minimum phase and Non mínimum phase.

The initial conditions for the state vector and control input voltages are specified as follows

$$x_k = [\,0, 0, 0, 0\,]^T$$

$$u_k = \begin{cases} \begin{pmatrix} [\,3.00, 3.00]^T [V] - \text{MP} \\ [\,3.15, 3.15]^T\ [V] - \text{NMP} \end{pmatrix} 0 \le k < 750; \\ (\,[\,1.00, 1.00]^T\ [V] - \text{MP \&NMP})\ 750 \le k < 1750; \\ \begin{pmatrix} [\,3.00, 3.00]^T\ [V] - \text{MP} \\ [\,3.15, 3.15]^T [V] - \text{NMP} \end{pmatrix} 1750 \le k < 2100; \\ (\,[\,1.00, 1.00]^T\ [V] - \text{MP \&NMP}\,)\ 2100 \le k < 2400; \\ \begin{pmatrix} [\,3.00, 3.00]^T\ [V] - \text{MP} \\ [\,3.15, 3.15]^T\ [V] - \text{NMP} \end{pmatrix} 2400 \le k < 2750; \\ (\,[\,1.00, 1.00]^T\ [V] - \text{MP \&NMP})\ 2750 \le k \le 3500; \end{cases}$$

The other parameters are considered constant and are kept the same as their steady state values shown in Table 1.

The measurable states of the QTP are the liquid level in tanks 1 and 2 (x_1 and x_2). These measurements are used in the observer correction step to update the estimated states recursively. The ELO observer first utilizes the nonlinear model of QTP (5) for prediction and corrects the estimates using a constant gain matrix K (13). Specifically, the state estimate is updated by adding the correction term $K(z_k - \hat{z}_k)$, where z_k represents the measurement outputs.

In contrast, Extended Kalman Filter operates with a recursive prediction - correction structure. At each time step, the predicted state vector (15) and error covariance (16) are computed based on the system dynamics. The Kalman gain K_k is computed (17). With this gain, the state estimates (18) and covariance matrix (19) are corrected using the latest measurements. The process (Q_k) and measurement (R_k) noise covariance matrices are selected as $0.01 * I_4$ and $0.01 * I_2$ to ensure accurate tracking of tank levels under these conditions.

Figures 8 and 10 show the ELO and EKF estimated states in the QTP system for the change in pump voltage 1 and 2 for the NMP and MP configuration. When comparing the system's actual states to its predicted state, the state estimation error is the variation between the two. Figures 9 and 11 display the state estimation error for the QTP Non-Minimum Phase system and the Minimum Phase system, respectively.

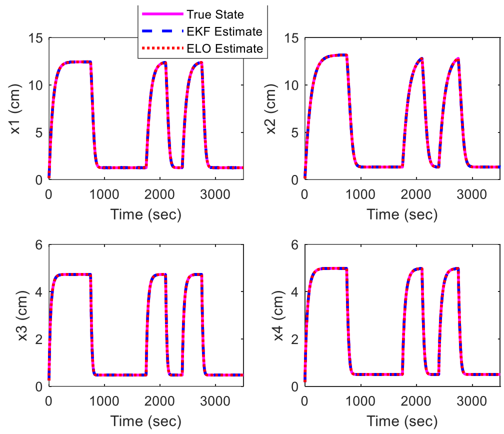

Fig. 8. Normal Conditions- True and Estimated levels of NMP QTP.

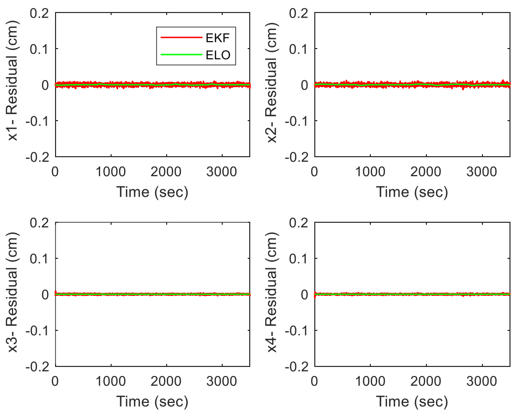

Fig. 9. Normal Conditions - State estimation error of NMP QTP.

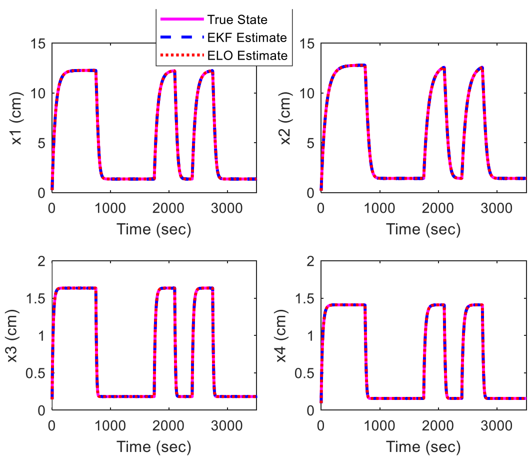

Fig. 10. Normal Conditions - True and Estimated levels of MP QTP.

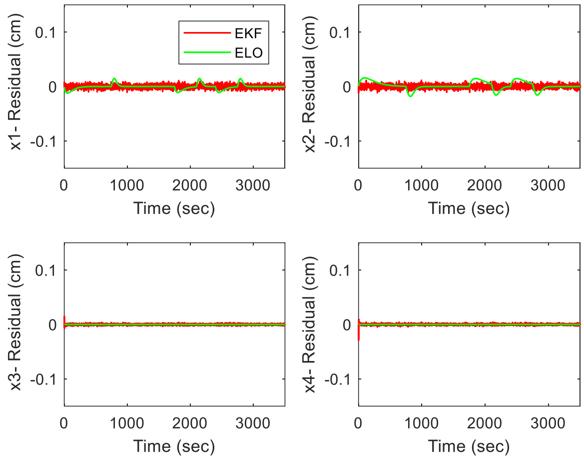

Fig. 11. Normal Conditions - State estimation error of MP QTP.

Scenario 3: Process Uncertainty

In this scenario, the parameter variations in the quadruple tank system are considered. It is important to note that the state vector initialization, input voltage and observer procedures are kept identical to the normal conditions earlier. The simulations have been conducted for MP and NMP configurations of the quadruple tank system incorporating process uncertainties. Since the drain of tank 1 (a_1) and tank 3 (a_3) significant impact the system dynamics, variations in these parameters are treated as sources of process uncertainty in this work. These uncertainties may arise due to salt deposition, sedimentation or clogging in the outlet pipes, which effectively alter the cross-sectional area. Uncertainty levels of 10%, 20%, and 30% are introduced into QTP to evaluate the robustness of ELO and EKF algorithms.

Figures 12, 14 and 17 show that the EKF, ELO state algorithms estimated the liquid levels of QTP under 10%, 20% and 30% process uncertainty respectively, for the non-minimum phase configuration. Figures 13, 15 and 18 illustrate the state estimation error under 10%, 20% and 30% process uncertainties of the quadruple tank NMP system.

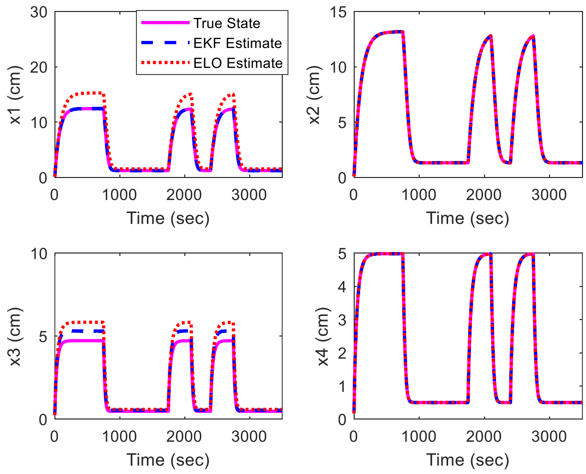

Fig. 12. True and the estimated states of NMP QTP using EKF, ELO under 10% process uncertainty.

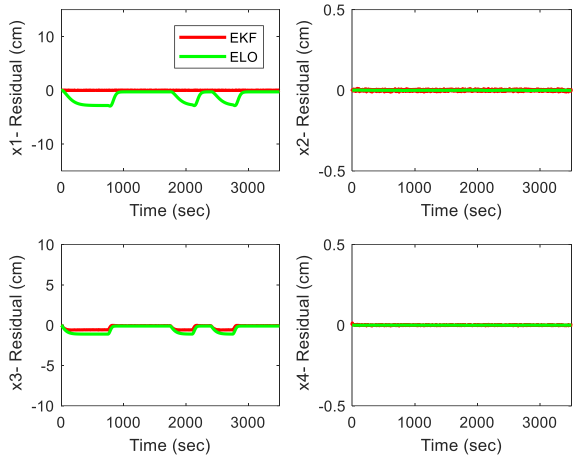

Fig. 13. NMP - State estimation error - 10% process uncertainty.

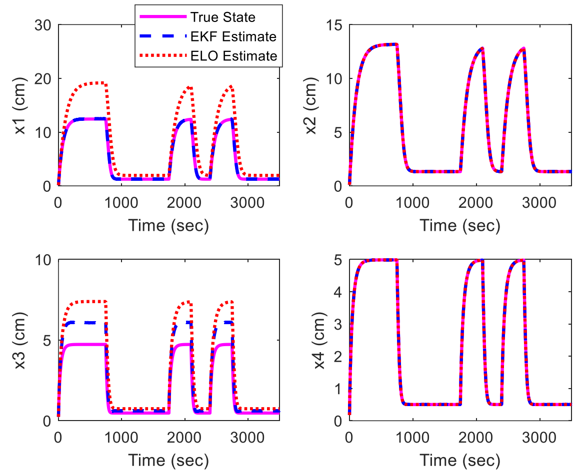

Fig. 14 . True and the estimated states of NMP QTP using EKF, ELO under 20% process uncertainty.

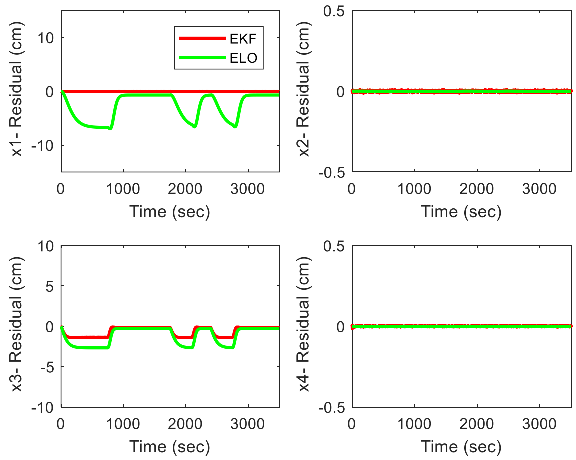

Fig. 15. NMP- State estimation error - 20% process uncertainty.

The gain matrix of the Extended Kalman Filter and Extended Luenberger Observer is represented as a 4*2 matrix. The gain of both state estimators under 30% uncertainty in the Non-Minimum Phase of the QTP are illustrated in Figure 16. From the figure, it is inferred that ELO employs a constant gain throughout the operation. In contrast, the EKF gain varies recursively at each time step. This variation of the EKF gain is noticeable at the input voltage changes occurring at 750s, 1750s, 2100s, 2400s and 2750s, especially for states x_3 and x_4. The adaptive gain adjustment enables the EKF to track the system states more accurately than ELO, resulting in small estimation errors.

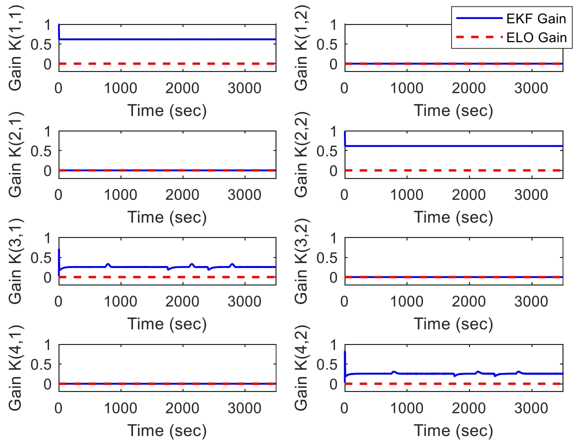

Fig. 16. EKF and ELO gain values for NMP QTP under 30% process uncertainty.

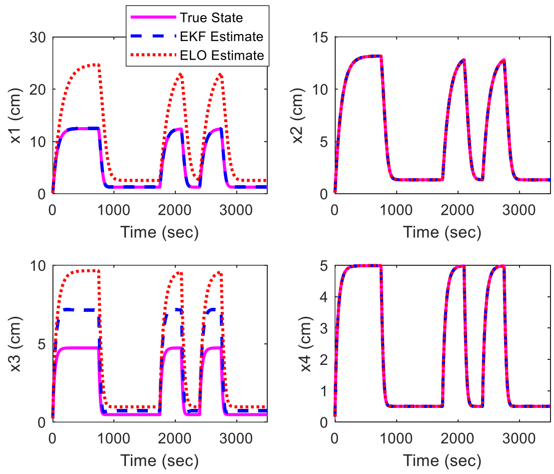

Fig. 17. True and the estimated states of NMP QTP using EKF, ELO under 30% process uncertainty.

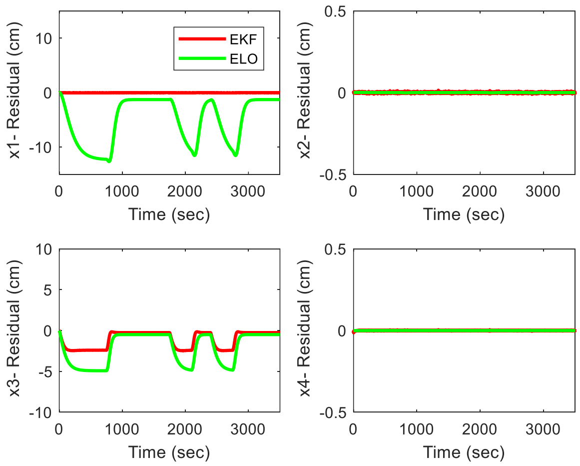

Fig. 18. NMP- State estimation error - 30% process uncertainty.

Figures 19, 21, and 23 present the true and estimated liquid levels of Minimum Phase quadruple tank system under 10%, 20%, and 30%, uncertainty respectively. Figures 20, 22 and 24 display the corresponding state estimation errors.

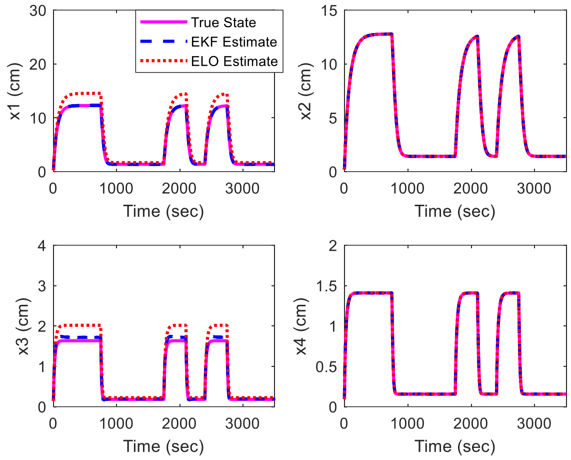

Fig. 19. True and the estimated states of MP QTP using EKF, ELO under 10% process uncertainty.

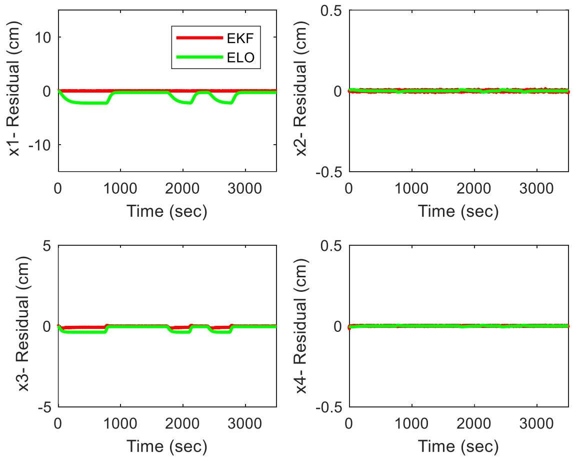

Fig. 20. MP- State estimation error - 10% process uncertainty.

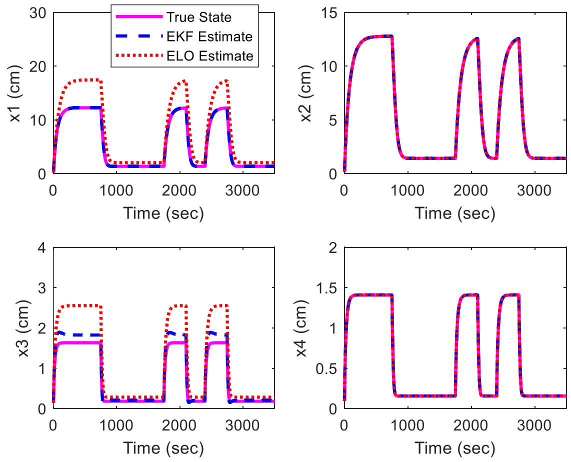

Fig. 21. True and the estimated states of MP QTP using EKF, ELO for 20% process uncertainty.

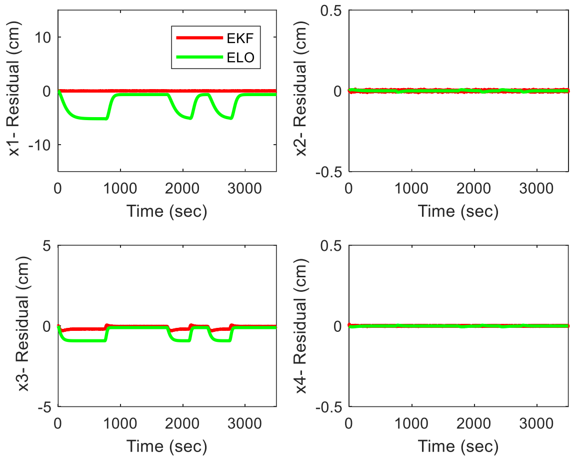

Fig. 22. MP- State estimation error 20% process uncertainty.

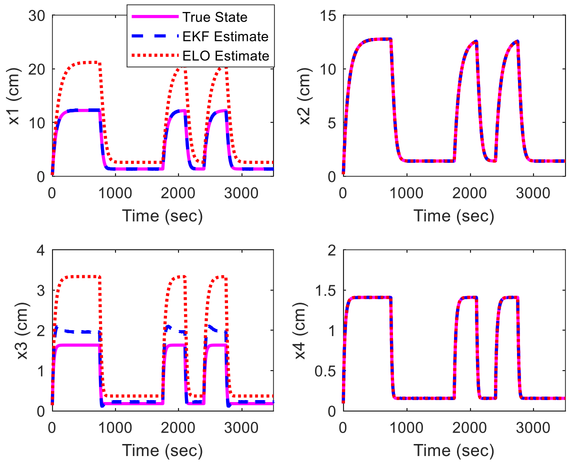

Fig. 23. True and the estimated states of MP QTP using EKF, ELO for 30% process uncertainty.

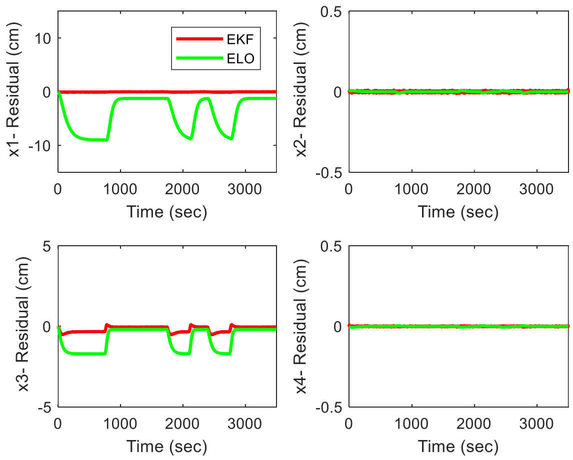

Fig. 24. MP- State estimation error 30% process uncertainty.

4.2 Discussion

The performance metrics MSE, ISE and IAE are computed to analyze the state estimator effectiveness. A detailed comparison for normal condition, as presented in Table 2 (refer to Annex 1), reveals that ELO shows better estimation performance than EKF for the Non Minimum Phase configuration, with lower MSE values. Although the EKF also performs well, it exhibits slightly higher estimation error than the ELO in this configuration. In contrast, for Minimum phase configuration of the QTP, the EKF performs better than the ELO under normal conditions. It shows lower MSE values for Tanks 1 and 2, highlighting its effectiveness in dynamic environments.

Table 3 (refer to Annex 1) summarized the NMP configuration of the QTP performance evaluation measure under uncertainty. It shows that the EKF achieves lower MSE values for the states x_1 and x_3 than ELO. Conversely, the ELO performs better for the states x_2 and x_4 , for those less affected by the introduced uncertainty and ELO shows poor performance at all uncertainty levels.

Table 4 (refer to Annex 1) presents the comparison of EKF and ELO for the MP configuration of QTP based on error metrics under the conditions of 10%, 20% and 30% process uncertainties. It shows that the EKF achieves lower MSE values for all states affected by uncertainty, compared to ELO in the minimum phase configurations. The ELO performance deteriorates significantly at all uncertainty levels. The performance of the EKF has remained strong and consistent with low MSE values for 10%, 20% and 30 % uncertainty level in MP configuration.

Based on the evidence presented, it can be deduced that the EKF can deliver precise estimates that maintain smaller error metrics in the presence of uncertainty. Furthermore, ELO's performance continued to worsen, exhibiting significant errors and instability, which brought to light the system's limitations when subjected to high levels of uncertainty. The analysis of ISE and IAE is similar to MSE.

From a practical stand point, estimation errors in ELO poses significant risk during prolonged operation. In real time, these errors may lead to incorrect control actions, actuator overcompensation, and degradation of system components, potentially compromise system stability and safety. In contrast, EKF demonstrates superior performance against uncertainties, consistently maintaining lower error values. This robustness arises from its recursive update mechanism and adaptability through Jacobian-based linearization. EKF may exhibit minor errors, there are typically short-lived and manageable. Overall EKF proves more suitable for dynamic, uncertain environments. On the other hand, ELO due to its less computational complexity, is better suitable for systems with stable dynamics.

The superior performance of the EKF under nonlinear dynamics can be associated its ability to linearize the non-linear system around an estimated trajectory using Jacobian matrices. This iterative linearization allows the EKF to adapt to system changes more effectively than ELO, while using a fixed gain matrix and it is less flexible in adapting to sudden or nonlinear changes in QTP behavior. The recursive nature of EKF enables it to minimize the error metrics in state estimation by incorporating prediction and correction approach, which is beneficial in nonlinear system. ELO is easier to implement with fewer adjustment, but EKF require careful tuning and initialization. Computational complexity is lower for ELO but in EKF higher computational complexity due to matrix inversions and updates. Providing a detailed comparison of EKF and ELO in MP and NMP configurations under various conditions, this dual configuration analysis of nonlinear state estimator design helps bridge gaps in the literature that often focus on singular configuration of linear state estimator. By highlighting the strengths and weakness of the EKF and ELO in handling real world conditions, ELO can achieve lower state estimation error in ideal environments. EKF offers greater robustness, making it the preferred choice for uncertain scenarios.

The demonstrated state estimation of the EKF under the various level of process uncertainties indicate its suitability for applications, where the system parameter may not be perfectly known or subject to change. This includes chemical processing plants, distillation column and any industrial applications involving interconnected multi tank configurations.

5. CONCLUSION

This work presented a comparative study of the Extended Kalman Filter and Extended Luenberger Observer for the state estimation of the nonlinear quadruple tank system. The simulation was carried out to estimate the level of four tanks under normal plant and process uncertainties in the MP and NMP configurations of QTP. Under normal conditions, EKF outperformed ELO in MP configuration and ELO achieved superior estimation accuracy for NMP configuration. However, in process uncertainties, the EKF consistently outperforms ELO in terms of maintaining low performance metrics (MSE, ISE and IAE), particularly in states directly affected by the uncertainty. These findings indicate that nonlinear state estimator ELO is preferable for simple or ideal operating conditions, while EKF is better suited for uncertain, dynamic and nonlinear environment. Specifically, EKF offers better robustness and adaptability to model uncertainties, making it more suitable for further control and fault detection applications in complex process systems like the QTP. Future work will focus on integrating these into fault detection and fault-tolerant control (FTC) for MP and NMP configurations of the quadruple tank system.

REFERENCES

Alhajeri, M. S., Wu, Z., Rincon, D., Albalawi, F., & Christofides, P. D. (2021). Machine-learning-based state estimation and predictive control of nonlinear processes. *Chemical Engineering Research and Design*, 167, 268-280.

Aljuwaiser, E., Badr, R., & Hassan, M. F. (2018). A new approach of state estimation of linear discrete systems. *Journal of Control Engineering and Applied Informatics*, 20(2), 3-13.

Azam, S. N. (2017). Linear discrete-time state space realization of a modified quadruple tank system with state estimation using Kalman filter. *In Journal of Physics: Conference Series, IOP Publishing*, 783(1), 012013.

Chandra, K. P. B., & Gu, D. W. (2019). *Nonlinear filtering*. Cham, Switzerland: Springer.

Chandra, K.P.B., Srikanthan, H., Gu, D.W., Bandyopadhyay, B., and Postlethwaite, I. (2012). Discrete-time sliding mode control using an h∞ filter for a quadruple tank system. *International Workshop on Variable Structure Systems*, 349-354.

Gurjar, B., Chaudhari, V., and Kurode, S. (2021). Parameter estimation based robust liquid level control of quadruple tank system-second order sliding mode approach. *Journal of Process Control*, 104, 1–10.

Hedengren, J. D., Shishavan, R. A., Powell, K. M., & Edgar, T. F. (2014). Nonlinear modeling, estimation and predictive control in APMonitor. *Computers & Chemical Engineering*, 70, 133-148.

Johansson, K. H. (2002). The quadruple-tank process: A multivariable laboratory process with an adjustable zero. *IEEE Transactions on control systems technology*, 8(3), 456-465.

Kalman, R.E. (1960). A new approach to linear filtering and prediction problems. *J. Basic Eng*, 82(1), 35-45.

Luenberger, D. (1971). An introduction to observers. *IEEE Transactions on automatic control*, 16(6), 596– 602.

Meng, X., Yu, H., Xu, T., & Wu, H. (2020). Disturbance observer and L 2-gain-based state error feedback linearization control for the quadruple-tank liquid-level system. *Energies,* 13(20), 5500.

Rigatos, G., Serpanos, D., Siadimas, V., Busawon, K., Gao, Z., Siano, P., & Abbaszadeh, M. (2018). Condition monitoring for the quadruple water tank system using H-infinity Kalman Filtering. *MATEC Web of Conferences, EDP Sciences,* 188, 05008.

Simon, D. (2006). *Optimal state estimation: Kalman, H infinity, and nonlinear approaches*. John Wiley & Sons.

Sim, X. W., Kek, S. L., Sim, S. Y., & Li, J. (2023). State estimation and optimal control of four-tank system with stochastic approximation approach. *Advances in technology innovation*, 8(2).

Smida, F., Säid, S. H., & M'Sahli, F. (2014). Unknown input observer-based flow rate and liquid level estimation for a quadruple tank process. *International Conference on Electrical Sciences and Technologies in Maghreb*, *IEEE,* 1-6.

Valarmathi, R., Muthukumar, N., Ramkumar, K., Balasubramanian, G., & Srinivasan, S. (2015). State estimation of Quadruple tank system using nonlinear estimator. *International Conference on Computation of Power, Energy, Information and Communication*, IEEE. 0513-0517.

Venkateswarlu, C. and Karri, R.R. (2022). *Optimal State Estimation for Process Monitoring, Fault Diagnosis and Control*. Elsevier.

Vijula, D. A., & Devarajan, N. (2013). Optimal regulator design using kalman's state estimator for a nonlinear multivariable process. *International Journal of Electrical Engineering and Technology*, 6(2), 149-163.

Wafi, M. K., & Widjiantoro, B. L. (2023). Distributed estimation with decentralized control for quadruple-tank process. *arXiv preprint arXiv:2304.04763*.

Welch, G., Bishop, G., et al. (1995). *An introduction to the kalman filter*.

Zhou, M., Cao, Z., Wang, J., & Wang, C. (2020). Interval state and sensor fault estimation based on unknown input observer and interval hull computation. *The Canadian Journal of Chemical Engineering*, 98(6), 1339-1350.

Appendix A. LIST OF ACRONYMS

CV - Control Valve
EKF- Extended Kalman Filter
ELO- Extended Luenberger Observer
IAE- Integral Absolute Error
ISE- Integral Square Error
MIMO - Multi Input Multi Output
MP- Minimum Phase
MSE- Mean Square Error
NMP- Non Mínimum Phase
QTP- Quadruple Tank Process
UKF- Unscented Kalman Filter

ANNEX 1

Table 2. Performance metrics (MSE, ISE and IAE) of state variables for the MP and NMP configuration of the QTP under ideal conditions.

Performance Measure	states	Non-Minimum Phase		Minimum Phase	
		EKF	ELO	EKF	ELO
MSE	x_1	1.1266e-05	3.2726e-07	1.1649e-05	2.1434e-05
	x_2	1.1316e-05	5.3409e-07	1.0843e-05	5.7701e-05
	x_3	1.8744e-06	0	1.8216e-06	0
	x_4	1.9957e-06	4.7484e-15	1.5760e-06	6.6177e-12
ISE	x_1	3.9443e-02	1.1457e-03	4.0781e-02	7.5041e-02
	x_2	3.9617e-02	1.8699e-03	3.7962e-02	2.0201e-01
	x_3	6.5621e-03	0	6.3774e-03	0
	x_4	6.9870e-03	1.6624e-11	5.5177e-03	2.3168e-08
IAE	x_1	9.3966	1.1298	9.4582	8.6100
	x_2	9.4550	1.6992	9.2342	1.7762e+01
	x_3	3.6968	0	3.498	0
	x_4	3.8178	1.5052e-04	3.4778	5.2822e-03

Table 3. Performance metrics (MSE, ISE and IAE) of state variables for the non-minimum phase configuration of the QTP under 10%, 20% and 30% process uncertainty.

Performance Measure	States	EKF			ELO		
		10%	20%	30%	10%	20%	30 %
MSE	x_1	1.4491e-04	5.5596e-04	1.2479e-03	2.5919e+00	1.4076e+01	4.4362e+01
	x_2	1.1417e-05	1.1174e-05	1.1308e-05	5.3800e-07	5.3703e-07	5.3442e-07
	x_3	1.3469e-01	7.2071e-01	2.2659e+00	4.4758e-01	2.5119e+00	8.2890e+00
	x_4	1.8221e-06	1.8100e-06	2.0168e-06	4.7836e-15	4.7780e-15	4.7488e-15
ISE	x_1	0.5073	1.9464	4.3688	9074.1086	49280.5841	155311.2248
	x_2	0.0400	0.0391	0.0396	0.0019	0.0019	0.0019
	x_3	471.5667	2523.2222	7932.8280	1566.9733	8794.3316	29019.7588

	x_4	0.0064	0.0063	0.0071	0.0000	0.0000	0.0000
IAE	x_1	36.0118	71.9798	108.6631	4254.8300	10000.5127	18005.8249
	x_2	9.4781	9.3647	9.3377	1.7120	1.7081	1.6968
	x_3	948.0315	2193.0699	3890.7183	1744.0922	4149.6344	7591.7273
	x_4	3.7433	3.7239	3.7448	0.0002	0.0002	0.0002

Table 4. Performance metrics (MSE, ISE and IAE) of state variables for the minimum phase configuration of the QTP under 10%, 20% and 30% process uncertainty.

Performance Measure	**States**	**EKF**			**ELO**		
		10%	**20%**	**30%**	**10%**	**20%**	**30%**
MSE	x_1	1.4686e-04	5.5411e-04	1.2261e-03	1.8205e+00	9.3216e+00	2.7344e+01
	x_2	1.0843e-05	1.1055e-05	1.1553e-05	5.7701e-05	5.8045e-05	5.7301e-05
	x_3	3.8563e-03	1.9469e-02	5.7537e-02	5.7122e-02	3.2414e-01	1.0874e+00
	x_4	1.5760e-06	1.5928e-06	1.7049e-06	6.6177e-12	6.6531e-12	6.5928e-12
ISE	x_1	0.5142	1.9399	4.2926	6373.6422	32634.9608	95732.8476
	x_2	0.0380	0.0387	0.0404	0.2020	0.2032	0.2006
	x_3	13.5010	68.1627	201.4355	199.9844	1134.8178	3806.9763
	x_4	0.0055	0.0056	0.0060	0.0000	0.0000	0.0000
IAE	x_1	36.5118	72.9678	109.0379	3668.8969	8371.3466	14506.8270
	x_2	9.2342	9.2199	9.5256	17.7624	17.8557	17.7026
	x_3	161.0942	360.4464	617.0655	624.4552	1490.8962	2740.9775
	x_4	3.4778	3.4792	3.5747	0.0053	0.0053	0.0053