

Optimizing Wind Energy Capture: A Comparative Analysis of Instantaneous and Average Wind Speed Controls in Wind Turbines

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Abstract: This study investigates the effects of fluctuating wind speeds on the efficiency of energy capture by wind turbines, focusing on a 2.5 MW turbine operating in Dobrogea, Romania. The study contrasts two control strategies: one using average wind speed measurements and the other based on instantaneous wind speed data. By leveraging experimental observations, the research highlights the superiority of instantaneous measurements in adapting turbine operations to real-time wind conditions. A mathematical framework is employed to elucidate the dynamics of wind energy conversion, emphasizing the necessity for continuous adjustment of turbine parameters. Findings demonstrate that accurately accounting for wind speed variability is crucial for optimizing energy production, with implications for the advancement of control systems in wind energy technology.

Keywords: Wind turbine control, instantaneous wind speed, average wind speed, energy efficiency, renewable energy systems.

1. INTRODUCTION

Wind energy has become a cornerstone of global efforts to transition toward renewable energy systems. Maximizing the efficiency of wind turbines, especially in regions with rich wind resources such as Dobrogea, Romania, is essential for improving the viability and sustainability of this energy source. However, the unpredictable and variable nature of wind speeds presents unique challenges for turbine operations, necessitating innovative control strategies to ensure maximum energy capture.

Previous research has played a crucial role in advancing wind turbine optimization. For example, the maximum power point tracking algorithms designed for fluctuating wind conditions were investigated, demonstrating their potential to improve turbine efficiency. Likewise, Wang et al. (2021) developed advanced control systems that leverage real-time data to enhance energy production efficiency. Neural network-based control strategies have also proven effective, as demonstrated in (Anderson and Wei, 2020), while the analysis in (Patel and Choudhary, 2021) shed light on hybrid energy systems integrating wind and solar technologies. These studies form the foundation for the present investigation.

Additional advancements in the field have further informed this research. For example, the study in (Garcia and Patel, 2022) analyzed turbulence modeling techniques that enhance turbine performance under fluctuating wind conditions. In (Gupta and Ramesh, 2020) insights into wind speed forecasting methods, which are essential for proactive turbine adjustments, are provided.

Recent innovations in predictive analytics and machine learning have also contributed to the development of more effective control strategies. For instance, the integration of predictive modeling into wind turbine optimization is discussed in (Kim et al., 2023), while the potential of machine learning algorithms to enhance energy capture is demonstrated in (Zhang & Sun, 2021). These advancements underscore the importance of incorporating cutting-edge technologies into turbine control systems.

Achieving optimal energy capture relies on effective Maximum Power Point Tracking (MPPT) strategies that adapt to fluctuating wind conditions. A novel MPPT control method based on the rotor speed Probability Density Function (PDF) improves response time and stability for enhanced power extraction (Zhang & Wang, 2022).

Optimizing torque control strategies further stabilizes power output and reduces mechanical stress, improving turbine efficiency (Smith & Adams, 2023). Additionally, Super-Twisting Sliding Mode Control (ST-SMC) offers robust stability for DC microgrids, mitigating power fluctuations and ensuring effective energy management in distributed systems (Lin et al., 2023).

Recent advancements emphasize adaptive and intelligent control techniques designed to manage these conditions effectively. Xu et al. (2023) introduced adaptive control methods, dynamically adjusting turbine parameters in real-time for improved energy capture. Chen and Kumar (2024) extended this by applying deep reinforcement learning to Maximum Power Point Tracking (MPPT), significantly outperforming traditional control methods. Ali et al. (2023) utilized predictive control frameworks, demonstrating

efficiency improvements under dynamic wind conditions. Johnson and Patel (2024) further refined this approach using Model Predictive Control (MPC), effectively enhancing both efficiency and structural stability during turbulent conditions.

Building on this body of knowledge, the current study evaluates the effectiveness of control strategies based on instantaneous versus average wind speed measurements. By integrating theoretical modeling with empirical data from a 2.5 MW turbine operating in Dobrogea, this research aims to quantify the impact of fluctuating wind speeds on turbine performance and identify the optimal approaches for maximizing energy capture. The results offer practical insights for optimizing the design and operation of wind turbines across various environmental conditions.

2. WIND TURBINE'S MATHEMATICAL MODEL

Wind turbine operates in a dynamic regime when the wind speeds fluctuate (Carstea et al., 2020). Hence, the equation governing the kinetic motion is employed:

$$J \frac{d\Omega}{dt} = T_{WT} - T_{EG} \quad (1)$$

where: Ω – is the generator's speed, J – is the total inertia,

T_{WT} – is the wind turbine's electromagnetic torque provided by the wind turbine, T_{EG} – is the generator's electromagnetic torque.

Multiplying (1) by Ω , the following power equation is obtained:

$$J \frac{d\Omega}{dt} \Omega = P_{WT} - P_{EG} \quad (2)$$

where: P_{WT} – is the wind turbine's output power, P_{EG} – is the generator's output power.

In modern systems, the generator load (P_{EG}) is adjusted based on the wind speed value, considering the relationship between the generator's optimal speed and wind speed, as described by Huynh et al. (2021), and Singh Pali et al. (2020). If this requirement is not fulfilled, the maximum potential of wind energy cannot be captured. To maximize the energy captured by a wind turbine at a given wind speed, the turbine must denoted as Ω_{opt} or n_{opt} (Carstea et al., 2020).

Maximizing wind turbine performance by operating at its maximum power point (MPP) and optimal speed (Ω_{opt}) is a challenging task that is influenced by fluctuating wind speeds and wind turbine's total inertia. Figure 1 illustrates the functioning of a wind turbine at its optimal rotational speed (Ω_{opt}) and maximum power point (MPP). It shows how the load on the electric generator changes with different wind speeds (v). The shaded red area represents the region of optimal energy efficiency, which is determined by the wind turbine's power characteristics ($P_{WT}(\Omega)$) corresponding to (v_1) and (v_2) wind speeds. The $P_{WT}(\Omega)$ function attains its highest value at MPP. For constant wind speeds, it is easy to estimate the maximum power output. However, when wind

speeds vary over time, this maximum power output changes from MPP_1 to MPP_2 , depending on the wind speed variations.

In this current case, an increase in wind speed from v_1 to v_2 , leads to a range of wind turbine power characteristics, illustrated by the hatched area in Figure 1.

$$P_{WT}(\Omega, 4.09) = 1543 \cdot 10^4 \left(\frac{4.09}{\Omega} - 2.63 \cdot 10^{-2} \right) e^{-58.39 \left(\frac{4.09}{\Omega} \right)} 4.09^3 \quad (3)$$

$$P_{WT}(\Omega, 4.28) = 1543 \cdot 10^4 \left(\frac{4.28}{\Omega} - 2.63 \cdot 10^{-2} \right) e^{-58.39 \left(\frac{4.28}{\Omega} \right)} 4.28^3 \quad (4)$$

Optimal control of wind turbines in the presence of highly fluctuating wind speeds is challenging and often ineffective due to the difficulty in accurately determining the optimal speed value (Ω_{opt}), which varies depending on the wind speed values and, therefore needs to be constantly recalculated to target the energetically optimal range. This adjustment is essential due to variations in atmospheric conditions, mechanical constraints, and changes in the turbine's geometry over time.

Apart from integrating the wind turbine operation into the MPP at Ω_{opt} , capturing wind energy is another difficult problem, as mentioned in (Carstea et al., 2020). In the majority of the literature papers (Garcia & Patel, 2022; Gupta & Ramesh, 2020), all these challenges are addressed considering constant wind speed and no optimal wind speed target.

From the specialized literature, the wind turbine power corresponding to the speed (Ω) is calculated using the relation:

$$P_{WT} = \rho \pi R_p^2 C_p(\lambda) v^3 \quad (5)$$

where: ρ - is the air density in the wind turbine's operating location; R_p – is the blades radius; $C_p(\lambda)$ – is the power conversion factor; $\lambda = \Omega R_p / v$; v – is the wind speed.

The power conversion factor ($C_p(\lambda)$) is determined by the relationship (Carstea et al., 2020):

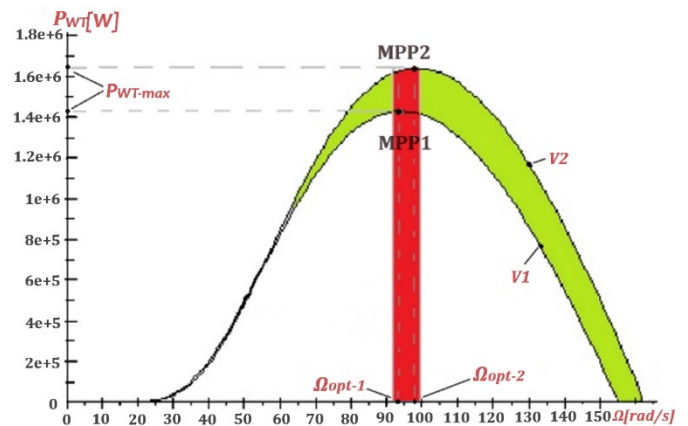


Figure 1. Wind turbine's power characteristics ($P_{WT}(\Omega)$)

$$C_p(\lambda) = C_1 \left(\frac{C_2}{\lambda} - C_3 \right) e^{-\frac{C_4}{\lambda}} \quad (6)$$

where: C_1, C_2, C_3, C_4 are constructive constants given in the catalog.

$$\frac{1}{\lambda} = \frac{1}{\lambda} - 0.03 = \frac{v}{R_p \Omega} - 0.03 = \left(\frac{v}{1.5\Omega} - 0.03 \right) \quad (7)$$

By substituting λ , the power conversion factor ($C_p(\lambda)$) is expressed in the form:

$$\begin{aligned} C_p(\lambda) &= C_1 \left(\frac{C_2}{\lambda} - C_3 \right) e^{-\frac{C_4}{\lambda}} \\ &= C_1 \left(C_2 \left(\frac{v}{1.5\Omega} - 0.03 \right) - C_3 \right) e^{-C_4 \left(\frac{v}{1.5\Omega} - 0.03 \right)} \\ &= a_1 \left(\frac{v}{\Omega} - b \right) e^{-c \frac{v}{\Omega}} \end{aligned} \quad (8)$$

Finally, the wind turbine's power is calculated using the relation:

$$P_{WT}(\Omega, v) = a \left(\frac{v}{\Omega} - b \right) e^{-c \left(\frac{v}{\Omega} \right)} v^3 \quad (9)$$

where the parameter values a, b and c can be determined from the experimental data.

The value corresponding to the maximum power is obtained by setting the derivative of the power equal to zero:

$$\frac{dP_{WT}}{d\Omega} = \frac{d}{d\Omega} \left(a \left(\frac{v}{\Omega} - b \right) e^{-c \left(\frac{v}{\Omega} \right)} v^3 \right) = 0 \quad (10)$$

and the ideal rotational speed is determined as:

$$\Omega_{opt} = \frac{c}{1} + bc \cdot v = k_{\Omega} \cdot v \quad (11)$$

Therefore, the value of the report: $\frac{v}{\Omega} = \frac{v}{\Omega_{opt}} = 1 + \frac{bc}{c}$

replaced in (9) will provide the value of the maximum power, corresponding to the optimal mechanical angular speed (Ω_{opt}):

$$\begin{aligned} P_{WT_max}(v) &= a \left(\frac{v}{\Omega} - b \right) e^{-c \left(\frac{v}{\Omega} \right)} v^3 \\ &= \frac{a}{c} e^{-1-cbv^3} = k_1 v^3 \end{aligned} \quad (12)$$

The maximum power can also be determined using the mechanical speed value:

$$P_{WT_max}(\Omega) = k_2 \Omega^3 \quad (13)$$

There is a linear relationship between the optimal speed (Ω_{opt}) and the wind speed (v), as demonstrated. The wind turbine's mathematical model is determined using experimental data obtained from the 2.5 [MW] WT type: GEWE-B2.5-100, which is currently operating in the

Dobrogea area, specifically in the Galbiori location (Carstea et al., 2020). The data, retrieved on 10/20/2023, includes the measured wind speed, wind turbine's actual speed, and generator's power, as shown in Appendix, based on that, values of the parameters a, b and c are determined as follows:

1. the value of parameter b is derived from the maximum mechanical angular speed (Ω_{max}) due to the absence of wind turbine power.

$$P_{WT} = a \left(\frac{v}{\Omega} - b \right) e^{-c \left(\frac{v}{\Omega} \right)} v^3 = 0 \quad (14)$$

$$\text{at: } \Omega_{max} = \frac{18}{8} \Omega_{opt} = \frac{18}{8} 154.99 = 348.73 \left[\frac{rad}{s} \right] \quad (15)$$

$$\text{Yielding to: } \frac{v}{\Omega_{max}} - b = 0 \quad (16)$$

By replacing v and Ω_{max} with their numerical values the parameter b is obtained:

$$b = \frac{v}{\Omega_{max}} = \frac{10.77}{348.73} = 3.08 \cdot 10^{-2} \quad (17)$$

2. the value of parameter c is determined based on the optimal speed value (Ω_{opt}) that corresponds to the maximum power. This optimal value is obtained by setting the derivative of power to zero:

$$\frac{dP_{WT}}{d\Omega} = \frac{d}{d\Omega} \left(a \left(\frac{v}{\Omega} - b \right) e^{-c \left(\frac{v}{\Omega} \right)} v^3 \right) = 0 \quad (18)$$

$$\text{or } \Omega_{opt} = \frac{c}{1+bc} v \quad (19)$$

As the value of parameter b is known, then the value of parameter c is obtained:

$$\frac{\Omega_{opt}}{v - \Omega_{max} \cdot b} = \frac{154.99}{10.77 - 154.99 \cdot 3.08 \cdot 10^{-2}} = 25.90 \quad (20)$$

3. the value of parameter a is obtained from the wind turbine's maximum power value in (13), corresponding to the optimal speed value (Ω_{opt}):

$$\begin{aligned} a &= \frac{P_{WT_max}}{v^3} e^{1+cb} c = \frac{2243850}{10.77^3} e^{1+0.79 \cdot 25.9} \\ &= 2.81 \cdot 10^5 \end{aligned} \quad (21)$$

In conclusion, the wind turbine's mathematical model is obtained (Carstea, C., et al., 2020):

$$\begin{aligned} P_{WT_max} &= 2.81 \cdot 10^5 \left(\frac{v}{\Omega} - 3.08 \right. \\ &\quad \left. \cdot 10^{-2} \right) e^{-25.9 \left(\frac{v}{\Omega} \right)} v^3 \end{aligned} \quad (22)$$

Verification: at time instant 17.44 [s], optimal wind turbine's speed value is $\Omega_{opt} = 154.99$ [rad/s] and, corresponding to it, the wind speed is $v = 10.77$ [m/s]. Same value was also obtained from measurements.

The actual speed value (Ω_{real}) is obtained from the real speed (n):

$$\Omega_{real} = \frac{\pi}{30} n \quad (23)$$

The defined optimal speed for mechanical systems is:

$$\Omega_{opt} = k_{\Omega} v \quad (24)$$

The optimal mechanical speed (Ω_{opt}) must adjust according to the wind speed (v), resulting in the function $\Omega_{opt}(t)$, from Figure 2. Also, $\Omega_{real}(t)$ is obtained from the measured values of the speed. In the ideal case, the two graphs overlap, which is not the case in Figure 2, where differences between the two graphs are large, reaching up to:

$$\frac{0.10 \cdot 1458 - 14.39 \cdot 7.14}{14.39 \cdot 7.14} 100 = 48.48\% \quad (25)$$

which corresponds to time instant $t^* = 43.073$.

At present, the wind speed and generator's power have the following values:

$$v^* = 7.72 \text{ [m/s]} \quad (26)$$

$$P_{EG} = 1945.84 \text{ [kW]} \quad (27)$$

The actual mechanical speed has the value:

$$\Omega_{real}(43.07) = 0.10 \cdot 1458 = 152.68 \text{ [rad/s]} \quad (28)$$

while the optimal mechanical speed has the value of:

$$\Omega_{opt}(43.07) = 0.10 \cdot 1458 = 111.1 \text{ [rad/s]} \quad (29)$$

The wind turbine's maximum power can be calculated based on the ideal mechanical speed.

$$P_{WT_max}(\Omega) = 0.6\Omega^3 = 0.6 \cdot 111.1^3 = 8.26 \cdot 10^5 \text{ [W]} \quad (30)$$

At time instant 17.44 [s], the turbine is operating at maximum power point (MPP) and optimal speed $n_{opt} = 1480$ [rpm].

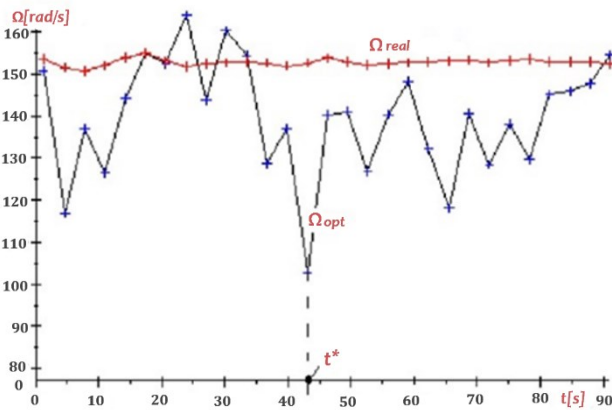


Figure 2. Fluctuations in the wind turbine's optimal (Ω_{opt}) and actual (Ω_{real}) speeds

Therefore, the optimal mechanical speed is calculated as:

$$\Omega_{opt} = 1480 \frac{\pi}{30} = 154.99 \text{ [rad/s]} \quad (31)$$

Since at time instant 17.44 [s], the measured wind speed is $v = 10.77$ [m/s], the linear relationship between the optimal mechanical speed and wind speed leads to the following result:

$$\Omega_{opt} = 154.99 = k_{\Omega} v = k_{\Omega} 10.77 \quad (32)$$

Which yields to: $k_{\Omega} = 14.391$. This way, the optimal mechanical speed can be calculated from the wind speed, with the relationship:

$$\Omega_{opt} = 14.391v \quad (33)$$

Thus, maximum power of the generator, measured at time instant 17.44 [s] is:

$$P_{EG} = P_{WT_max} = 22438.5 \text{ [kW]} \quad (34)$$

$$\text{And from: } P_{WT_max}(\Omega) = k_2 \Omega^3 \quad (35)$$

By numerical replacement $k_2 = 0.6$.

$$\text{Thus, (35) becomes: } P_{WT_max}(\Omega) = 0.6\Omega^3 \quad (36)$$

Same results can be also obtained by using the following:

$$\frac{v}{\Omega_{opt}} = \frac{10.77}{154.99} = 6.94 \cdot 10^{-2} \quad (37)$$

Then, the above equation is replaced in (22), resulting the wind turbine's maximum power:

$$P_{WT_max} = 2.81 \cdot 10^5 (6.94 \cdot 10^{-2} - 3.08 \cdot 10^{-2}) e^{-25.90(6.94 \cdot 10^{-2})} v^3 = 1796.2v \quad (38)$$

And, by replacing v with $\Omega_{opt} = 14.39v$, the wind turbine's maximum power is obtained:

$$P_{WT_max}(\Omega, v) = 1796.2 \left(\frac{\Omega_{opt}}{14.39} \right)^3 = 0.6\Omega_{opt}^3 \quad (39)$$

Remark 1. The maximum power output of the wind turbine is influenced by the cube of either the optimal mechanical speed (Ω_{opt}) or the wind speed (v^3). This relationship underscores the importance of situating wind turbines in regions with consistently high wind speeds to maximize energy generation.

In conclusion, the mathematical model developed for the wind turbine in this study accounts for key factors such as air density, blade radius, and wind speed, ensuring a comprehensive representation of its performance dynamics. Key equations governing turbine operation are derived to estimate optimal mechanical angular speed and power output. Experimental data from the turbine's operation in Dobrogea inform the parameterization of the model, ensuring its applicability to real-world conditions.

3. COMPARATIVE STUDY OF THE WIND ENERGY CAPTURED FROM TWO POINTS OF VIEW: INSTANTANEOUS AND AVERAGE WIND SPEEDS

When operating a wind turbine under time-varying wind speeds, the question arises as to whether its optimal control should be based on instantaneous wind speed values or on average wind speed values.

This decision is crucial for maximizing the amount of wind energy that the turbine can capture. Since the wind speed directly influences the energy captured by the turbine, determining the most effective control strategy is essential for optimizing performance.

The wind turbine is regulated according to the Ω_{opt} , which is directly correlated with the fluctuating wind speed.

Direct measurements can be utilized to evaluate the time variation of wind speed from which both the instantaneous and average values result.

For instance, let's consider a 2.5 [MW] wind turbine GEWE-B2.5-100 (Carstea et al., 2020).

In this case, the time interval is 10 [min], and therefore Ω_{opt} is calculated at a sampling step of 10 [min].

The correlation of this value depends on both the wind speed derivative ($\frac{d\Omega}{dt}$) and the total inertia (J).

J primarily dictates the time interval over which the average wind speed is calculated, influencing the turbine's dynamic response to wind fluctuations.

The wind turbine captures a larger amount of wind energy (E_{WT}) than the energy produced by the generator (E_{EG}).

The efficiency can be modified according to the values of both instantaneous and average speed.

Subsequently, the energy produced by the generator and the wind energy captured by the wind turbine will be calculated for two scenarios: using both instantaneous and average speed measurements.

3.1 Estimation of the energy introduced into the power grid

The electric generator power can be integrated to obtain the electrical energy injected into grid:

$$E_{EG} = \int P_{EG} dt \quad (40)$$

The analysis is conducted for two distinct cases: one where the wind speed fluctuates over time and another where the wind speed remains constant.

From the perspective of the captured wind energy, an ideal variation in wind speed is permitted in order to quantify the difference between the instantaneous and average wind speed values. This variation does not affect the validity of the conclusions drawn from the simulations.

3.1.1 Control system based on the instantaneous wind speed value

A sinusoidal variation of wind speed over time is considered in the analysis:

$$\begin{aligned} v(t) &= 10 + 1.4 \sin\left(\frac{2\pi}{100} t\right) \\ &= 10 + 1.4 \sin(6.28 \cdot 10^{-2} t) \end{aligned} \quad (41)$$

This sinusoidal wind speed variation (41) was selected for its mathematical tractability in demonstrating control concepts. Future work could incorporate the Weibull-distributed wind profiles or Monte Carlo simulations to assess performance across turbulence intensities.

From the wind turbine's mathematical model, the optimal speed (Ω_{opt}) yields as:

$$\Omega_{opt} = 14.39v = 14.39[10 + 1.4 \sin(6.28 \cdot 10^{-2} t)] \quad (42)$$

The ideal rotational velocity is achieved by adjusting the generator's power through the utilization of a PI (proportional integrator) type regulator, which is characterized by:

$$P_{EG} - P_{EG_{opt}} = k(\Omega - \Omega_{opt}) + \frac{k}{T_i} \int (\Omega - \Omega_{opt}) dt \quad (43)$$

which by derivation, yields to:

$$\frac{dP_{EG}}{dt} - \frac{dP_{EG_{opt}}}{dt} = k \frac{d\Omega}{dt} - k \frac{d\Omega_{opt}}{dt} + \frac{k}{T_i} (\Omega - \Omega_{opt}) \quad (44)$$

where $P_{EG_{opt}}$ – is the optimal value of the generator's power, k – is the amplification factor of the regulator, T_i – is the regulator's integration constant.

The PI (Proportional-Integral) regulator was chosen for its simplicity, reliability, and ease of implementation. It requires fewer computational resources compared to more complex control strategies like adaptive, fuzzy, or sliding mode controls. While these alternatives offer benefits in handling non-linearity and uncertainty, they are more complex and require extensive tuning or computational power, making the PI regulator a practical and efficient choice for this study.

It offers a straightforward approach to regulating turbine performance by minimizing steady-state error and ensuring stable operation. The proportional term helps in reducing the error by adjusting the control signal based on the current deviation, while the integral term eliminates steady-state errors over time.

The PI regulator, with the error ($\Omega - \Omega_{opt}$) as its input, determines the switching angle of the thyristors in the converter positioned between the generator and the grid.

From (2) and (44) the wind system's equations are obtained:

$$\begin{cases} J \frac{d\Omega}{dt} \Omega = P_{WT} - P_{EG} \\ \frac{dP_{EG}}{dt} - \frac{dP_{EG_{opt}}}{dt} = k \frac{d\Omega}{dt} - k \frac{d\Omega_{opt}}{dt} + \frac{k}{T_i} (\Omega - \Omega_{opt}) \end{cases} \quad (45)$$

The initial conditions are:

$$\begin{cases} P_{EG}(0) = 1.2 \cdot 10^6 < P_{WT}(0) = P_{EG_{opt}}(0) = 1.79 \cdot 10^6 \\ \omega(0) = 122 < \Omega_{opt}(0) = 143.91 \end{cases} \quad (46)$$

The amplification factor of the regulator can be obtained from:

$$P_{EG}(0) - P_{EG_{opt}} = k (\Omega(0) - \Omega_{opt}(0)) \quad (47)$$

The solution is: $k = 27211$.

Tuning of the regulator supposes a complex task due to the system's non-linearity and it can only be achieved through successive simulations, finally yielding to $T_i = 5$.

Therefore, the wind system's equations become:

$$\begin{cases} 511.92 \frac{d\Omega}{dt} \Omega = 2.81 \cdot 10^5 \left(\frac{10 + 1.4 \sin 6.28 \cdot 10^{-2} t}{\Omega} - 3.08 \cdot 10^{-2} \right) \cdot e^{-25.9 \frac{10 + 1.4 \sin 6.28 \cdot 10^{-2} t}{\Omega}} \cdot (10 + 1.4 \sin 6.28 \cdot 10^{-2} t)^3 - P_{EG} \\ \frac{dP_{EG}}{dt} = 27211 \frac{d\Omega}{dt} - 27211(1.26 \cos 6.28 \cdot 10^{-2} t) + \frac{27211}{5} (\Omega - 143.9(10 + 1.4 \sin 6.28 \cdot 10^{-2} t)) \\ \frac{dE_{EG}}{dt} = P_{EG} \end{cases} \quad (48)$$

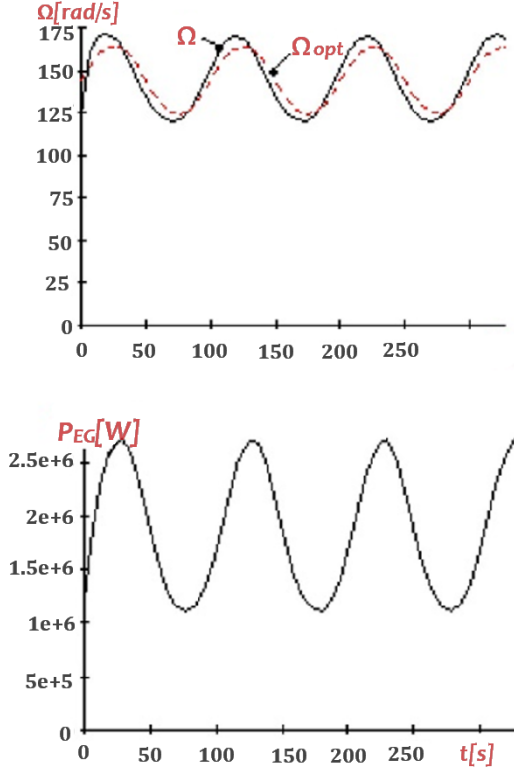


Figure 3. Fluctuations of the wind turbine's speed and generator's power for variable wind speed $v \neq ct$.

The electric energy injected into the grid over the time interval of 300 [s] is $E(300) = 5.5 \cdot 10^8$ [J].

With the PI regulator properly tuned, the wind turbine operates within the MPP region, while its actual speed gradually approaches the optimal speed, as illustrated in Figure 3.

3.1.2 Control system based on the average wind speed value

Considering that the wind speed is constant in time, for a given time interval, the wind turbine control, in this case, is based on the average value of the actual speed and, therefore, the reference value of the regulator (Ω_{opt}) is constant.

It value yields from the average speed from the sinusoidal variation over time of the wind speed: $v(t) = 10$ [m/s].

In this case, (50) becomes:

$$\begin{cases} 511.92 \frac{d\Omega}{dt} \Omega = 2.81 \cdot 10^5 \left(\frac{10}{\Omega} - 3.08 \cdot 10^{-2} \right) \cdot e^{-25.9 \frac{10}{\Omega}} 10^3 - P_{EG} \\ \frac{dP_{EG}}{dt} = 27211 \frac{d\Omega}{dt} + \frac{27211}{5} (\Omega - 143.9) \\ \frac{dE_{EG}}{dt} = P_{EG} \end{cases} \quad (49)$$

For constant wind speed, $v(t) = 10$ [m/s], variations in time of wind turbine's speed and power are illustrated in Figure 4.

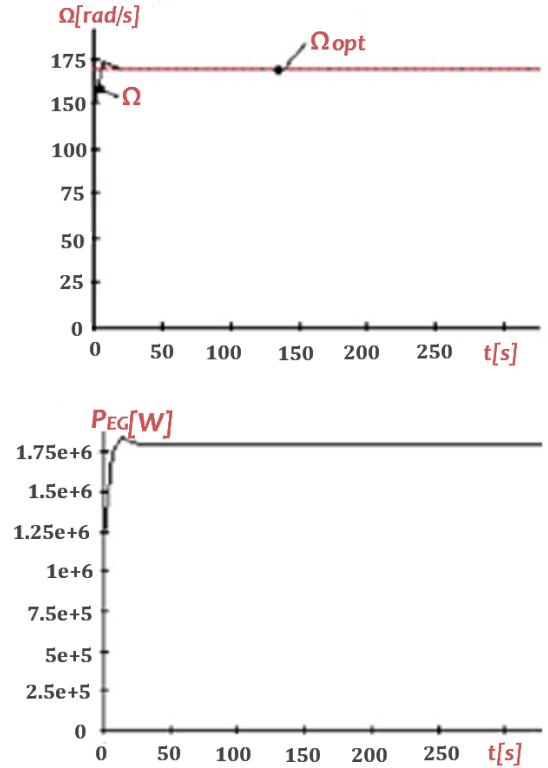


Figure 4. Fluctuations of the wind turbine's speed and generator's power for constant wind speed $v = ct$.

And, during the time interval of 300 [s], the energy injected into the grid is $E(300) = 5.37 \cdot 10^8 [J]$, which is smaller compared with the previous case when the wind speed was variable over time.

The difference between the two cases is 2.53%, which is quite a difference, considering the very high energy values of a

turbine in one year: $E(300) = 365 \cdot 24 \cdot 2 = 17520$ [MWh]/turbine/year. Therefore, the average value of the actual wind speed can be determined by analyzing the energy losses during the turbine's operation: $\Delta E = 17520 \cdot 2.53/100$ [MWh]/turbine/year.

3.2 Calculation of the captured wind energy

The output power of a wind turbine is determined by wind turbine's speed, which reaches a maximum value for optimal speed value Ω_{opt} :

$$P_{WT_max} = 2.81 \cdot 10^5 \left(\frac{v}{\Omega_{opt}} - 3.08 \cdot 10^{-2} \right) \cdot e^{-25.9 \left(\frac{v}{\Omega_{opt}} \right)} v^3 \quad (50)$$

When the wind turbine operates at (MPP) and Ω_{opt} , its performance is regulated by adjusting the generator's load according to variations in wind speed (Kim et al., 2023; Lin et al., 2023; Wang et al., 2021 and, Zhang & Sun, 2021).

The maximum power output of the wind turbine is obtained at:

$$\frac{v}{\Omega_{opt}} = \frac{1}{14.39} = 6.94 \cdot 10^{-2} \quad (51)$$

Yielding to:

$$P_{WT_max} = 2.81 \cdot 10^5 (6.94 \cdot 10^{-2} - 3.08 \cdot 10^{-2}) \cdot e^{-25.9(6.94 \cdot 10^{-2})} v^3 = 1796.2 v^3 \quad (52)$$

and based on it, by integration, the captured wind energy can be estimated:

$$E_{WT} = \int P_{WT} \cdot dt = 1796.2 \int v^3 \cdot dt \quad (53)$$

Next, the captured wind energy will be determined for two cases: based on instantaneous and average speed values.

3.2.1 Control system based on the instantaneous value of actual wind speed

For a sinusoidal variation of the wind speed, such as:

$$v(t) = (10 + 1.4 \sin 6.28 \cdot 10^{-2} t) \quad (54)$$

the value of $\frac{v}{\Omega_{opt}}$ can be obtained as:

$$\begin{aligned} \frac{v}{\Omega_{opt}} &= \frac{10 + 1.4 \sin 6.28 \cdot 10^{-2} t}{14.39 \cdot (10 + 1.4 \sin 6.28 \cdot 10^{-2} t)} \\ &= 6.94 \cdot 10^{-2} \end{aligned} \quad (55)$$

Thus, the maximum power output of the wind turbine results:

$$P_{WT_max} = 2.81 \cdot 10^5 \left(\frac{v}{\Omega_{opt}} - 3.08 \cdot 10^{-2} \right) \cdot e^{-25.903 \left(\frac{v}{\Omega_{opt}} \right)} v^3 = 1796.2 v^3 \quad (56)$$

And, then, the wind energy, captured over a time interval of 300 [s] can be calculated by integration:

$$\begin{aligned} E_{WT}(300) &= \int_0^{300} P_{WT_max} dt \\ &= \int_0^{300} (1796.2 (10 + 1.4 \sin 6.28 \cdot 10^{-2} t)^3) dt = 5.54 \cdot 10^8 [J] \end{aligned} \quad (57)$$

3.2.2 Control system based on the average value of actual wind speed.

In this case, the wind speed is assumed to remain constant over time, represented as the average of a sinusoidal variation of wind speed, $v(t) = 10$ [m/s]. Consequently, the energy captured over a time interval of 300 [s] is given by:

$$E_{WT}(300) = \int_0^{300} 1796.2 \cdot 10^3 dt = 5.38 \cdot 10^8 [J] \quad (58)$$

Then, it yields to a difference of 2.93% between the two system's control cases. Therefore, this difference can be found in the lost energy: $\Delta E = 17520 \cdot 2.93/100 = 515$ [MWh]/turbine/year, which is higher than in the previous paragraph, as it can be seen in Fig. 5.

The top graphic in Fig. 5 clearly shows the actual rotor speed closely tracking the optimal rotor speed, highlighting effective control performance despite wind fluctuations. The middle graphic illustrates the difference in generator power output between instantaneous and average wind-speed-based control methods. And, the bottom graphic highlights the exact 2.93% improvement in energy injection achieved by using instantaneous wind speed measurements.

Also, a clear comparative analysis table summarizing key metrics (rotor speed, generator power, and total energy injected) for both instantaneous and average wind speed control methods, including percentage improvements is shown below.

Table I.

Metric	Instantaneous wind speed	Average wind speed	Improvement [%]
Final rotor speed [rad/s]	150.90	143.84	4.91
Final generator power [MW]	1.78	1.80	-1.11
Total energy injected [MWh]/year /turbine	554.90	539.10	2.93

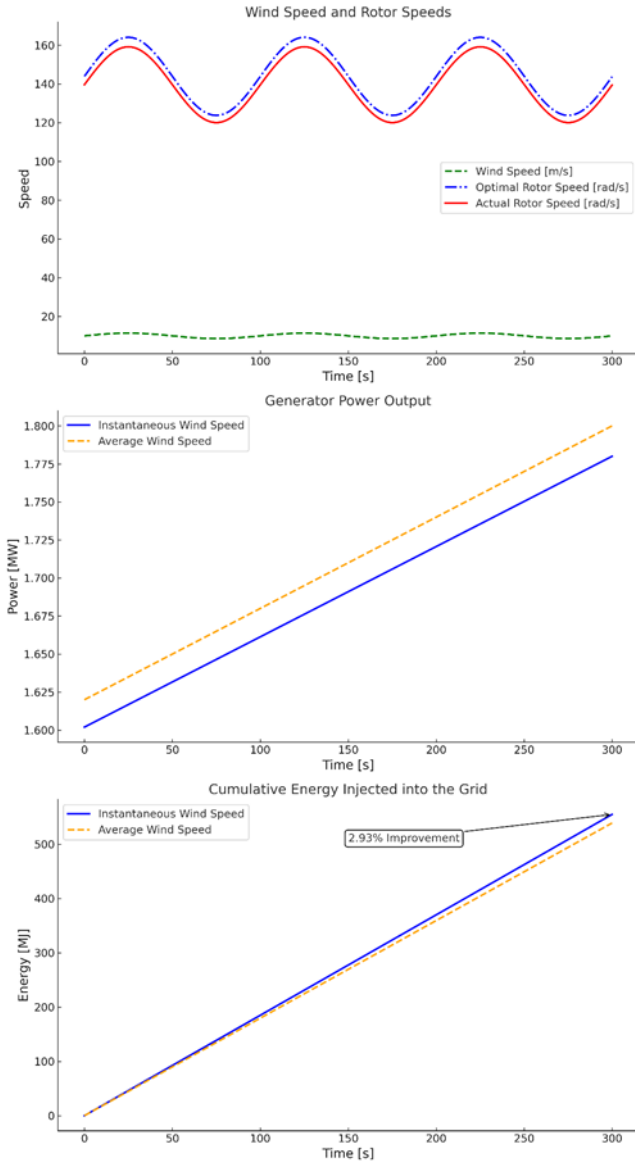


Figure 5. Cumulative energy injected into the grid versus generator power output versus wind speed and rotor speeds (instantaneous and average).

5. CONCLUSION & DISCUSSION

This study employed Scientific WorkPlace 5.5 for all mathematical modeling, equation solving, and numerical simulations. In order to achieve MPPT under fluctuating wind speeds, a combination of mathematical modeling and PI-based control was employed. Here's the breakdown:

1. Mathematical foundation:

- a dynamic model for a 2.5 MW wind turbine, incorporating real-world data (Appendix) from Dobrogea, Romania, to describe power output as a function of both wind speed (v^3) and optimal mechanical speed (Ω_{opt}^3) was developed
- the turbine power equation from real-world data was derived, accordingly to (12)

- the optimal mechanical speed (Ω_{opt}) as linearly dependent on wind speed (v) was established, as in (14)
- the MPPT was achieved when $\Omega = \Omega_{opt}$, maximizing P_{WT} .

2. Control strategy: A PI Regulator was employed by adjusted generator power (P_{EG}) to track Ω_{opt} in real-time, accordingly to (22). Its gains ($k = 27,211$; $T_i = 5$) were initially determined from steady-state conditions (26) and then through iterative refinement via simulation studies in order to minimize tracking error. However, the study could benefit in future from using established tuning methods such as Ziegler-Nichols or pole-placement techniques.

3. Validation: The field measurements from a 2.5 MW turbine in Dobrogea, Romania were used (Appendix).

The following contributions were achieved as a result of the present study:

- ✓ Determining a correct, real, wind turbine's mathematical model, is very useful in estimating important quantities in operation, such as:
 - the electrical energy fed into the power grid and the captured wind energy, adjusted based on the average or instantaneous wind speed.
 - the performance of a wind turbine in regions with energy-efficient characteristics, where wind speeds fluctuate over time.
 - the sample step value, which must be associated with the wind speed derivative and total inertia.
- ✓ The study compares the optimal and actual mechanical speeds over time highlighting significant variations that have an impact on wind energy capture.
- ✓ By tracking the operating point in the MPP, the optimal area was identified.
- ✓ Adjusting for average or instantaneous wind speed improved the usefulness of comparing the estimates of the generated energy and the captured wind energy. **Simulation results for energy capture were obtained for:** 1) instantaneous control by making real-time adjustments to $v(t)$ and, 2) average control by maintaining fixed Ω_{opt} based on mean wind speed. The results validate the advantages of instantaneous control, which yields a 2.93% increase in annual energy capture (equivalent to 515 [MWh]/year/ turbine).

The findings highlight the challenge of maintaining optimal turbine performance amidst fluctuating wind speeds, requiring real-time adjustments and accurate mathematical modeling in order to reflect changes in initial conditions such as air density, temperature, and blade position. Additionally, they emphasize the complexity of tuning control systems like PI regulators to ensure stable and efficient energy capture. Much more, to assure MPP operation, it is needed to specify a value for the generated power based on wind speed.

In conclusion, the presented results may also imply the need for further research to ensuring accurate and reliable sensor measurements to maintain optimal control performance. Additionally, managing computational demands and mitigating mechanical stresses caused by rapid control actions present significant practical difficulties.

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APPENDIX

$t[s]$	$V[m/s]$	Ω_{real} [rad/s]	P_{EG} [kW]
1.19	10.47	153.51	2277.83
4.63	8.13	151.63	2064.04
7.82	9.52	150.69	1813.47
11.04	8.8	152.05	1834.56
14.24	10.03	154.04	2024.8
17.44	10.77	154.98	2243.85
20.65	10.6	153.10	2185.28
23.85	11.39	151.73	2015.78
27.05	10	152.57	2076.22
30.24	11.14	152.78	2077.16
33.46	10.72	152.99	2037.22
36.66	8.94	152.26	2049.75
39.85	9.52	151.53	1977.82
43.07	7.145	152.26	1945.84
46.27	9.76	153.41	2080.21
49.47	9.81	152.57	2032.18
52.66	8.82	151.84	1977.47
55.88	9.76	152.05	1982.39
59.08	10.3	152.78	1954.04
62.27	9.2	152.89	1930.38
65.44	8.21	153.10	1977.94
68.69	9.77	153.41	2098.6
71.88	8.93	152.78	2024.92
75.08	9.6	153.20	2055.72
78.30	9.01	153.62	2117.22
81.49	10.09	152.99	2116.64
84.69	10.14	152.89	2159.04
87.91	10.27	152.99	2173.45
91.10	10.74	152.36	2058.18